WIRELESS ENGINEER

Vol. XXIV. MARCH 1947 No. 282

EDITORIAL

Alexander Graham Bell, born 3rd March, 1847—died 2nd August, 1922

ALEXANDER GRAHAM BELL was born in Edinburgh on 3rd March, 1847, the son of Alexander Melville Bell, who was a lecturer in elocution at New College, Edinburgh from 1843 to 1865, and whose father was also a prominent teacher of elocution in London and the author of several text-books on the subject. On the death of his father in 1865, Alex. Melville Bell moved from Edinburgh to London and became lecturer in elocution at University College. He was a noted authority on speech and published a great deal on vocal physiology, phonetics and visible speech ; i.e., a method of teaching people who were born deaf to speak. He visited Boston, Mass., in 1868, 1870 and 1871 to give courses of lectures.

His son Alexander Graham was eighteen when the family removed to London and he being then a resident teacher of elocution and music at Elgin, followed later and became a student at University College where his father was a lecturer. There had been three sons but the other two had died, and in 1870, as Alexander Graham's health was causing anxiety, the family removed to Canada and Alexander Melville Bell was appointed Professor of Elocution at Queen's College, Kingston, Ontario. After three years Alex. Graham's health was so improved that he accepted a position at the Boston School of Oratory. This was in 1873 and he was 26 years of age. He was an elocutionist of the third generation and his whole upbringing had been in an atmosphere of vocal physiology and phonetics. He had tried to determine the pitch of the tones that constitute the different vowels, and the resonant pitches of the mouth cavities when formed to utter the different vowels. When he wrote to a friend about this he was told that Helmholtz had already done this much more thoroughly. He then obtained a copy of Helmholtz's "Theory of Tone" and tried to repeat his experiments, but he confessed that at that time he was too slightly acquainted with the laws of electricity fully to understand the explanations given. He thought of the possibilities of producing music by electrically maintained tuning forks and of using a number of forks of different pitch for the simultaneous transmission of a number of messages over a single line by means of Morse Code. He studied the various systems of telegraphy then in use and became familiar with Morse keys, sounders, etc. He replaced the tuning forks by simple vibrating reeds which closed and opened the circuit, each reed at the receiving end only responding to the transmitting reed to which it was tuned ; at least, that was the intention.

One must not be misled by the use of the word telephone in accounts of these early experiments ; the transmission of a single tone by means of an electric current was sometimes referred to as electric telephony and when speech transmission

was attained it was sometimes referred to as articulate telephony.

Although Bell devoted himself wholeheartedly to the development of this harmonic telegraph, he was nursing the idea of the transmission of speech and proposed to his instrument maker and assistant, Watson, that he should make some instruments to try out the idea, but his future father-in-law, who was helping him financially, persuaded him to concentrate on the harmonic telegraph and not build such castles in the air. On June 2, 1875 a slight accident occurred that altered the whole course of events. Watson was in one room attending to the transmitting reeds while Bell in an adjacent room put each receiving reed in turn against his ear and adjusted the screw until it was properly tuned. Then the accident occurred, for the make-and-break points of one reed became welded together and Watson kept on plucking it to get it to restart. Bell rushed in to see what was happening, and as Watson said " the speaking telephone was born at that moment," for Bell saw that the peculiar sound that he had heard was not due to any intermittent current but to the undulatory current produced by the vibration of the steel spring over the pole of the magnet.

He told Watson to make a small drumhead of gold-beater's skin, with the centre joined to the free end of the steel spring and a mouth-piece over the drumhead. This was made at once and tested the next day with very little success. Although Watson could hear Bell's voice and almost catch the words, it required nine months of research before Watson heard a complete and intelligible sentence ; that was in March 1876, and during the summer such progress was made that, to use Watson's own words, " one didn't have to ask the other man to say it over again more than three or four times before one could understand quite well, if the sentences were simple." Bell seems to have done little more with the harmonic telegraph, which was perfected later by his rival Elisha Gray, who was also working on the transmission of speech and lodged a caveat for a patent on the same day that Bell applied for his patent, but an hour or two later.

By October 1876 the telephone was being tested between Boston and Cambridge a distance of two miles. The present type of receiver emerged as the result of hundreds of experiments with diaphragms varying from several feet in diameter to representations of the human ear, with electro-magnets and permanent magnets of all sizes and shapes. Bell used the same instrument as transmitter and receiver, but his patent application included a transmitter consisting of a diaphragm carrying a wire dipping into a cup of acidulated water or salt solution, thus varying the resistance as the diaphragm vibrated. This he exhibited together with the electromagnetic transmitter and receiver at the Centennial Exhibition at Philadelphia in 1876. One of the judges at this exhibition was Sir William Thomson, afterwards Lord Kelvin, and on his return he said " In the Canadian department I heard ' To be or not to be—there's the rub ' through an electric telegraph wire, but scorning monosyllables, the electric articulation rose to higher flights, and gave me messages taken at random from the New York newspapers. All this my own ears heard. This, the greatest by far of all the marvels of the electric telegraph, is due to a young countryman of my own, Mr. Graham Bell. Who can but admire the hardihood of invention which devised such very slight means to realize the mathematical conception that, if electricity is to convey all the delicacies of quality which distinguish articulate speech, the strength of the current must vary continuously and as nearly as may be in simple proportion to the velocity of a particle of the air constituting the sound." This was in 1876 when the instruments were in a very crude stage of development ; it shows that Kelvin fully appreciated the importance of Bell's insistence on an uninterrupted current made to vary in accordance with the particle velocity of the sound wave. It is interesting to note that the exhibit was in the Canadian department.

The first outdoor telephone line was installed in Boston in 1877, and at that time Bell gave a number of lectures in New York and in most of the large cities in New England, at which demonstrations were given of telephonic reproduction of distant singers and speakers. In August 1877 Bell married and made a trip to England ; on 31st October a special meeting of the Society of Telegraph Engineers was held in London at which Bell gave an address on " Researches in Electric Telephony." He concluded by saying that conversations had been carried on between New York and Boston, and that Mr. Preece, the Engineer-in-Chief to the Post Office, had informed him that conversations had

been successfully carried on through a submarine cable between Dartmouth and Guernsey. As an indication of the rapid appreciation of the value of Bell's work and of the development of the telephone on the commercial side, the Western Union Telegraph Co., declined the offer of Bell's father-in-law to sell them all the Bell patents for 100,000 dollars; this was early in 1876. Two years later, it was said that they would gladly have given 25,000,000 dollars for them.

Bell resigned his Professorship of Vocal Physiology at Boston and removed to Washington and it was at his house in Washington that his father Alex. Melville Bell died in 1905. Alexander Graham Bell died in Nova Scotia on 2nd August 1922.

G.W.O.H.

HIGH-SPEED WAVEGUIDE SWITCH*

By D. K. Bishop, B.Sc.

(Formerly, Radio Department, Royal Aircraft Establishment)

SUMMARY.—The high-speed switch described was developed to switch alternate pulses of r.f. power into the two aerials of a radar installation. This involves 500 switching operations per second. The device consists of a T-junction in a waveguide transmitting the H_{10} mode, the side arms being closed alternately by vanes on two discs rotated in synchronism with the pulses. It has been shown to operate satisfactorily between wavelengths of 9.8 cm and 10.2 cm and to handle a peak power of 500 kW.

1. Introduction

THE switch described was developed to fill a requirement in connection with a radar installation transmitting r.f. power in pulses of length 0.6 or 1.9 microseconds with a pulse recurrence frequency of 500 pulses per second at a wavelength within the band 9.8 cm to 10.2 cm. It was desired to switch alternate pulses to two aerial systems which were fed by a standard 3-inch by 1-inch rectangular waveguide, operating in the usual H_{10} mode. It was essential that the switch should remain open to a given aerial for a sufficient time to allow both transmission of a pulse and the reception of the reflected pulse from a target up to the maximum range of the equipment, since a common receiving-transmitting aerial was used. Also the switch must not present a voltage standing-wave ratio of more than about 1.3 to the transmitter, but since the pulse length was short (less than 2 microseconds) compared with the time between pulses (2,000 microseconds), the changeover, during which high standing-wave ratios may be produced, could be made to occur during the period between pulses when the transmitter was not working, by synchronizing the switch with the transmitter.

* MS. accepted by the Editor, June 1946.

2. Electrical Design

(a) *Junction*

Fig. 1 (*a*) represents an E-plane T-junction in a rectangular waveguide, with a conducting piston placed in one side arm at a distance x from the centre of the junction. It is found experimentally that if x and the dimensions of the junction are chosen suitably transmission of power into the other arm can take place without appreciable loss or mismatch.

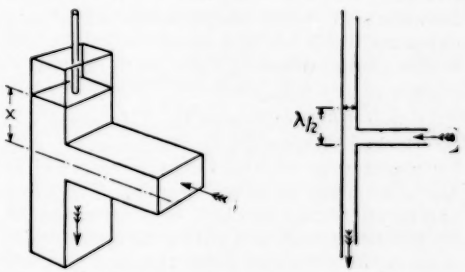


Fig. 1. *A T-junction and piston in a waveguide are shown at (a) and a junction in a two-wire transmission line at (b).*

Fig. 1 (*b*) shows an analogous arrangement in a two-wire transmission line, optimum power transfer round the angle occurs when

the series stub is short circuited at one half-wavelength from the junction, since this effectively replaces the stub by a short circuit at the junction. The line is then virtually continuous round the angle. In a rectangular waveguide an exact analogy is approached as the narrow dimension of the guide approaches zero. In this case a narrow waveguide (3 in by $\frac{1}{2}$ in) is used for mechanical reasons, and the correspondence between the stub length required (found experimentally to be 6.60 cm) and one-half of the guide wavelength (6.63 cm) is close.

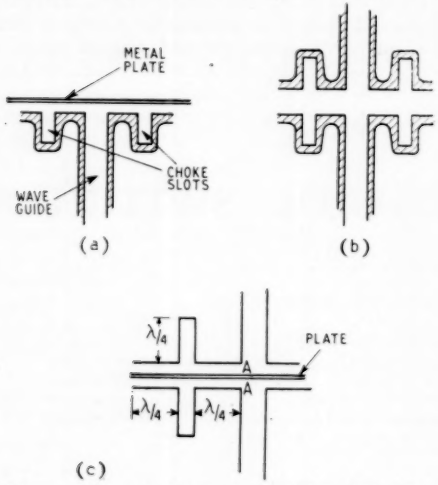


Fig. 2. *A waveguide can be blocked by fitting its open end with a flange having choke slots, and placing a conducting plate near it as at (a). With a double system (b) power is transmitted across the gap, but with the arrangement (c) the gap is blocked or open according to whether or not the plate is present.*

(b) Choke Flanges

If now the side arms of the T-junction can be blocked alternately at the correct points by a piston or its equivalent, power will be fed alternately to either branch. The guide can be effectively blocked, or short circuited, by fitting its open end with a flange provided with choke slots [see Figs. 2(*a*) and 3(*a*) and (*b*)] opposite which a metal plate is fixed at a small distance as in Fig. 2(*a*). With this distance as large as 0.4 cm it is found that an effective short circuit occurs at the flange and very little radiation takes place.

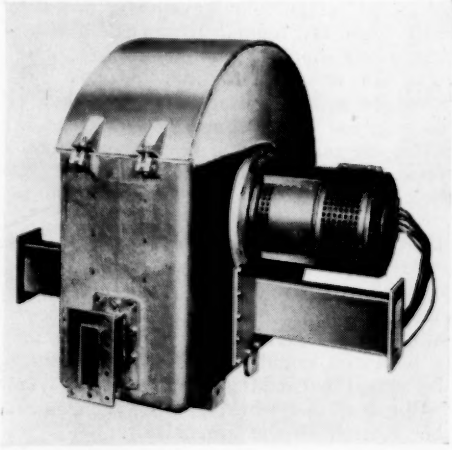
If a similar flange and waveguide are fixed opposite to it and the plate removed, Fig. 2(*b*), power is transmitted across the gap. When the dimensions are chosen, as in Fig. 2(*c*), the quarter-wavelength lines shown effectively produce a short circuit at A in each case, and the guide is either blocked or transmits power across the gap, depending on whether the plate is present or removed.

3. Switching

Switching can now be carried out by placing a metal plate in one gap while leaving the other open; this is done by two duralumin discs mounted on one spindle and rotating in the gaps between the choke flanges as in Fig. 3(*a*). The discs have alternate 36° sectors cut away, Fig. 3(*c*), so that when one end of the T-junction is closed by a metal sector the other disc presents a gap to the other end of the junction. Power is then transmitted into the open branch. The gap between the choke flanges is 0.60 cm and the discs approximately 0.3-cm thick, placed centrally so as to allow a gap of about 0.15 cm between each flange and the disc. Details of the choke flanges appear in Fig. 3(*b*).

4. Performance

The dimensions of the T-junction were determined experimentally by measuring the input standing-wave ratio of a T-junction having one side arm terminated by a choke flange and short-circuiting plate and the other by a matched load. Fig. 4 shows a curve of standing-wave ratio and wavelength for the final junction, having the dimensions given in Fig. 3.



General view of high-speed waveguide switch

Curves showing voltage-transmission coefficient and input standing-wave ratio (feeding matched loads) over a typical switching cycle at wavelengths of 9.8 cm, 10.0 cm, and 10.2 cm, are shown in Fig. 5.

diameter down to 17 inches. The switch is connected to the rest of the system by linearly tapered waveguides 8-in long, which are found to introduce no appreciable mismatch.

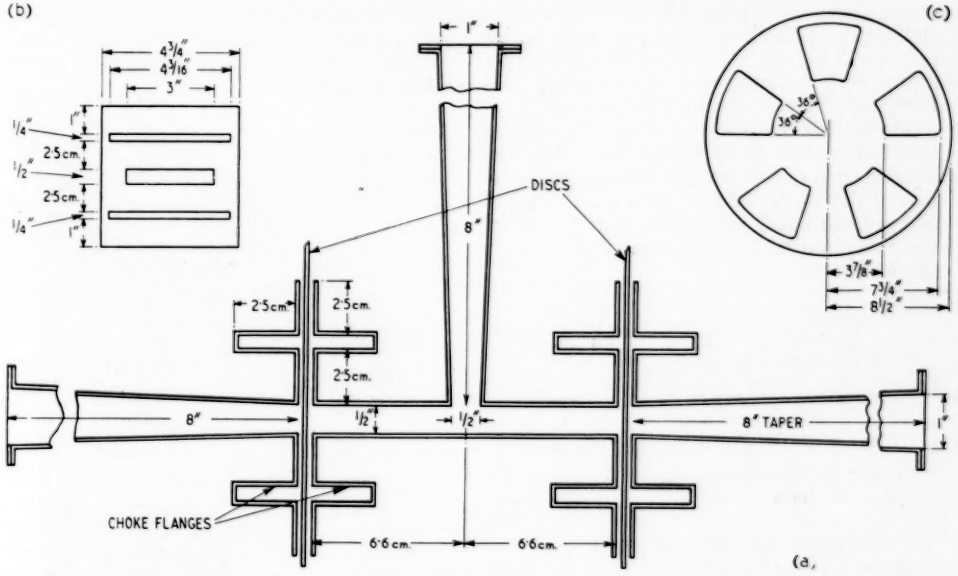


Fig. 3. *The general arrangement of the T-junction is shown at (a) with the details of the choke flanges and rotating discs at (b) and (c) respectively. Resonant dimension of waveguide 3" throughout.*

The difference in behaviour at the two switchings in each case is probably due to a misalignment in the prototype model, in which the vanes of one disc were not exactly opposite the gaps in the other.

Since the discs are rotated at a speed of 3,000 r.p.m., the 36° angle between switchings occupies 2,000 microseconds, of which about 1,550 microseconds are available as working time during which the standing-wave ratio remains below 1.25 and the transmission coefficient above 0.95. This time would enable a range of 150 miles to be obtained. Since the switching period, when the switch feeds both outputs and the standing-wave ratio is bad, occurs when the edges of the vanes cross the ends of the waveguides, and it is important to keep this time short, the angle subtended at the disc-centre by the width of the waveguide must be minimized. This can be done either by increasing the size of the discs or reducing the width of the guide. By using 3 in by $\frac{1}{2}$ in waveguide it is possible to keep the disc

The dimensions of the junctions and choke flanges are given in Fig. 3.

5. Drive and Phasing

The discs are directly coupled to a 180-volt 500-c/s synchronous motor having a speed of 3,000 r.p.m. corresponding to 500 switch throws per second. Since the 500-c/s supply is also used to trigger the transmitter pulse the switch can be phased to deliver a pulse alternately into each aerial.

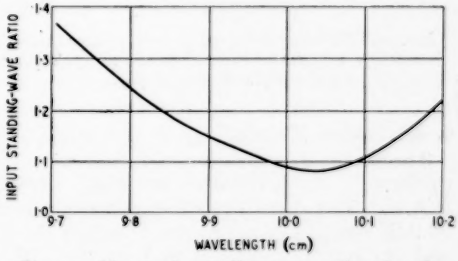


Fig. 4. *Measured standing-wave ratios for the junction of Fig. 3.*

Phasing is effected by rotating the motor relative to the switch housing, and the correct position is found by a stroboscopic device. A neon lamp is fitted inside the casing, where there is sufficient electric field to cause it to glow on each pulse. The edges of the vanes can then be seen as apparently stationary radial lines, and it is only necessary to adjust the motor until an edge comes in line with the lamp and a line marked on a Perspex window fitted to the housing.

The switch is adjusted so that after setting the motor the pulse comes on when the leading edge of a vane has passed the centre line of the waveguide by only 9°, the width of the vane being 36°. This allows time for the reception of the pulse reflected from a target.

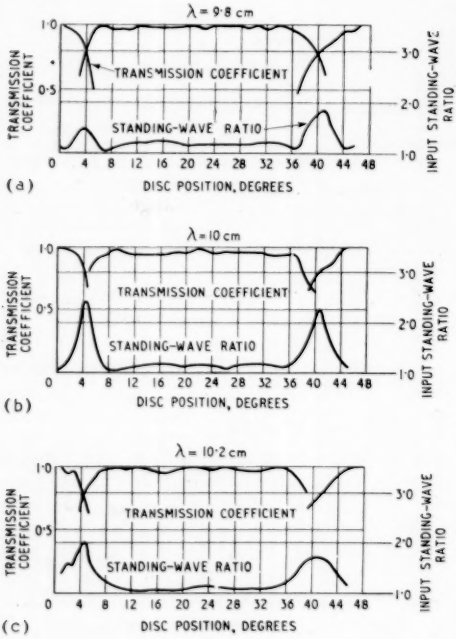
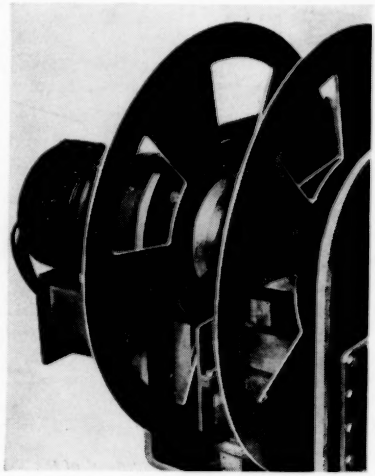


Fig. 5. *Standing-wave ratios and voltage-transmission coefficients over a switching cycle for three different wavelengths.*

6. Mechanical Details

The mechanical design of the switch is due to Mr. B. H. J. Rhodes of Radio Dept., R.A.E. The discs are of duralumin sheet (B.S.S. specn. L3) approximately $\frac{1}{8}$-in thick, and cut out as indicated in Fig. 3(*c*). The discs were planished, fitted with aluminium plates forming bosses, and mounted on a splined spindle. They were statically balanced after the bosses had been assembled. The disc spindle is carried on two ball bearings (B.S.S. spec. BR.L. 5/8) lubricated with a high melting-point grease. The clearance between discs and choke flanges is approximately 0.15 cm, the discs being central in the gap. This clearance has proved adequate in practice.



High-speed waveguide switch discs.

7. Power-handling Capacity

In radar technique, employing short pulses of power at comparatively long intervals, heating of waveguide components is of little importance as the mean power delivered is small, but during the pulse very high electric-field gradients are produced, which are liable to lead to breakdown and sparking; e.g., between the discs and the edges of the waveguide at the choke flanges. With the square-finished edges of flanges and vanes in the experimental model some trouble due to this effect was found, but was avoided by smoothing these edges to a small radius. The switch will handle a peak power of 500 kilowatts feeding a reasonably matched load and running synchronously.

Acknowledgments

The author wishes to acknowledge much valuable help and encouragement from Mr. J. L. Michiels and Mr. A. L. Cullen.

CAVITY RESONATORS AND ELECTRON BEAMS*

By J. H. Owen Harries, A.M.I.E.E., M.I.R.E.

(*Harries Thermionics Ltd., London*)

1. Introduction

DURING the past few years it became necessary, for the development of certain microwave valves, to analyse the conditions governing the transfer of energy from an electron beam into a resonant cavity, and from thence to a load, and to investigate the design of resonant cavities themselves. This paper describes the theoretical basis of the relationships involved, and the methods of measurement which were used.

An incidental, but important, result of the theoretical part of the programme is that a short-wave limit to present-day radio valve technique is shown to exist. It is at about 1-cm wavelength.

It is generally agreed nowadays that it is best to think and write about microwave technique in terms of classic electromagnetic field theory (Bib. 1), and not in terms of "lumped" or "distributed" circuit constants. This means using a set of mental concepts which are new to most radio engineers. It follows that papers on microwave phenomena should be presented in ways which assist the development of such concepts. Moreover, it always seems desirable to the author to write the principal equations in any engineering treatise in the most explanatory manner rather than—as seems fashionable nowadays—in their briefest forms; indeed, in this respect, one finds oneself following an illustrious precedent.†

The question arises as to what system of units it is best to employ. The metre/kilogram/second, or Giorgi, system has been adopted in this paper. Current and field integrals are measured respectively in coulombs/second (amperes) and volts/metre. The unit of resistance is the ohm. Distances and wavelengths are in metres. Wavelengths measured in metres are, perhaps, clumsy at the extreme microwave end of the radio gamut; but there is no reason why they should not be converted to centimetres in the final stages of computation.

Consider the problem of transferring energy from a modulated electron beam to an electromagnetic field in a resonant cavity, and, from this field, to a load. In this paper the words "electron beam" are not intended to mean only a narrow pencil of electrons; but will refer to any electron stream of definite shape, all the electrons in which have sensibly the same velocities at a given time and position along the beam path.

The beam must be caused to enter the cavity and must be modulated so that it will produce a rate of change of charge within the cavity. If the frequency of the modulation corresponds to a mode of resonance of the cavity, then an electromagnetic field consisting of "standing" waves will appear within it. A part of this field will extend into the conducting walls of the cavity, and will therefore dissipate energy in these walls. Another part of the field may be guided out of the cavity, and its energy may be either dissipated in other conductors, or radiated into space in the form of travelling electromagnetic waves. The electron beam may be modulated by apparatus external to the cavity resonator, or, alternatively, by energy withdrawn from the electromagnetic field within the cavity so that the device as a whole operates as a generator of oscillations.

2. Power Output Conditions

The phenomenon may be expressed quantitatively as follows:—

The electron beam may be looked upon as a rate of flow of charge—that is, as a current I_0 which enters the field inside the resonant cavity at an entrance velocity

$$v_0 = 5.95 \times 10^5 V_B^{\frac{1}{2}} \text{ metres/sec.} \quad \ldots \quad (1)$$

V_B may be looked upon as the entrance energy of each electron measured in volts,

* MS. accepted by the Editor, June 1946.

† Clerk Maxwell, (Bib. 1, Vol. II, Chap. IX, Art. 615).

LIST OF SYMBOLS

v_0 = the velocity at which an electron enters the resonant cavity in metres/sec.

V_B = the entrance energy of the electrons measured in electron-volts ($v_0 = 5.95 \times 10^5 V_B^{\frac{1}{2}}$ m/sec.)

I_0 = the electron beam current in amperes.

$\mathbf{v}$ = the volume of the resonator in metres^3.

$\mathbf{E}'$ = the maximum instantaneous value of the electric field in volts/metre.

$\mathbf{H}'$ = the maximum instantaneous value of the magnetic field in amperes/metre.

ϵ_0 = the specific inductive capacity of free space.

μ_0 = the permeability of free space.

W_F = the maximum instantaneous energy stored in a space in joules.

S = the energy loss per second in joules per unit area of a conducting surface.

μ_1 = the permeability of that surface.

σ_1 = the conductivity of that surface in mhos per metre.

H'_{tan} = the maximum instantaneous absolute value of the tangential component of $\mathbf{H}$ at that surface.

$\omega = 2\pi f$.

δ = the "skin depth" in metres.

λ = the wavelength in metres.

c = the velocity of light in metres/sec.

$\mathbf{s}$ = the internal surface area of the resonant cavity in metres.

P_R = the power loss in the walls of the resonator in watts/metre^2.

Q = the selectivity factor of an unloaded resonator.

Q_L = the selectivity factor of a loaded resonator.

P_0 = the power delivered from an electron beam to the resonator field in watts.

P_L = the power delivered to a load in watts.

η_c = the efficiency of the combination of a resonator and a load.

A = the cross-sectional area of the beam of electrons in metres.

i = the current density of the beam in amperes.

l_1 = the distance in metres along the length of the beam within the resonator field.

$E'l_1$ = the voltage along l_1.

V'_R = the electric field integral along l_1 in volts.

$$M = \frac{V'_R}{V_B}$$

ϕ = the "small signal" transit angle along l_1.

η_0 = the efficiency of transfer of power from the beam of electrons to the field of the resonator.

M_0 = the optimum value of M.

η = the overall efficiency of the electron beam-resonator-system.

ξ = the "voltage coefficient" of the resonator.

V_{OB} = the "characteristic voltage" of the resonator.

$$a = \frac{l_1}{\lambda}$$

A_0 = the area in metres^2 of the resonator which is utilized as the path of the beam of electrons along l_1.

Δ = the ratio between V_B and the current density i in the beam.

$$R_B = \frac{V_{OB}}{I_0}$$

ψ = the "resonator-beam ratio."

ζ = the "electronic coupling factor."

$k = A_0/\lambda^2$

λ_0 = a reference wavelength in metres (Q_0, R_{BO}, ψ_0, ξ_0 are the values of Q, R_B, ψ, and ξ at λ_0).

n, m, l = suffixes indicating the mode of resonance.

$r\ \phi, z$ = the co-ordinates of a cylindrical resonator.

r_0, z_0 = distances along these co-ordinates.

x, y, z = the co-ordinates of a rectangular box resonator.

x_0, y_0, z_0 = the distances in metres along these co-ordinates.

y_1, y_2 = distances in metres in the y-direction in a stepped rectangular box resonator.

J_0 = a Bessel function of the first kind and zero order.

r'_{nm} = the first (mth) root of $J_0(z) = 0$.

E'_z = the maximum value of electric field in the z-direction in a cylindrical resonator.

R = an integral due to Hansen & Richtmeyer.

$\Delta\lambda$ = the width of the resonance curve at 0.707 times the peak reading in the case of a linear indicator; or at 0.5 times the peak reading in the case of a square-law indicator.

Q_A, Q_B, Q_C = successive readings of Q.

E_1, E_2 = electric field values at different points in a resonator.

A_1, A_2 = the entrance and terminating areas of a tube of force.

and is approximately equal to the high tension voltage of the valve of which the resonant cavity is part. This current I_0 is modulated so that it possesses a characteristic wave-form. The fields produced by the resulting rate of change of charge inside the cavity will depend upon certain characteristics of the resonator.

The maximum instantaneous energy in an electromagnetic field inside the resonator is equal to both the maximum energy found in the magnetic field and to the maximum energy found in the electric field. When the energy in the electric field is at a maximum, that in the magnetic field is zero, and vice versa—as is always the case with "standing" waves. That is to say, the energy may be looked upon as existing in space and to have a maximum instantaneous value of

$$W_F = \frac{\epsilon_0}{2}\int \mathbf{E}'^2 . d\mathbf{v} = \frac{\mu_0}{2}\int \mathbf{H}'^2 . d\mathbf{v} \text{ (joules)} \quad (2)$$

where

$\mathbf{v}$ = the volume of the resonator.

$\mathbf{E}'$ = the maximum instantaneous value of the electric field, in volts/metre.

$\mathbf{H}'$ = the maximum instantaneous value of the magnetic field, in amperes/metre.

ϵ_0 = the specific inductive capacity of free space. This is equal to $10^{-9}/36\pi \approx 8.854 \times 10^{-12}$ farad/meter.

μ_0 = the permeability of free space. This is equal to 1.247×10^{-6} henry-meter.

The electromagnetic field will extend into the conducting walls of the cavity, and energy will therefore be dissipated in those walls. It may be shown (for instance, Bib. 2, p. 141) that the energy loss in joules per unit area (in metres^2) per second of a conducting surface may be found from the Poynting vector to be

$$S = \frac{1}{2}\sqrt{\frac{\mu_1\omega}{2\sigma_1}}\, H'^2_{tan} \quad .. \quad .. \quad (3)$$

where

μ_1 = the permeability of the surface (for copper this may be taken as equal to μ_0).

σ' = the conductivity of the surface (for copper this may be taken as 5.8×10^7 mhos/metre).

H'_{tan} = maximum instantaneous absolute value of the tangential component of the magnetic field at the surface of the resonator.

$\omega = 2\pi f$.

The power loss per period will then be S/f, or

$$\frac{2\pi}{\omega}\cdot\frac{1}{2}\sqrt{\frac{\mu_1\omega}{2\sigma_1}}\, H'^2_{tan} = \pi\sqrt{\frac{2}{\mu_1\omega\sigma_1}}\,\frac{1}{2}\mu_1 H'^2_{tan}$$

It can also be shown that the field may be looked upon as penetrating into the conducting surface for a distance

$$\delta = \sqrt{\frac{2}{\mu_1\omega\sigma_1}} \text{ (metres)} \quad .. \quad .. \quad (4)$$

This is usually referred to as the "skin-depth," and for copper

$$\delta = 3.8 \times 10^{-6}\lambda^{\frac{1}{2}} \text{ (metres)} \quad .. \quad (4.1)$$

where

λ = the wavelength in metres,

δ is plotted for copper in Fig. 1.

Thus, the power loss per metre^2 of the walls of a resonator will be equal at each point on its surface to

$$\frac{\pi c\delta}{2\lambda}\,\mu_1 H'^2_{tan} \text{ watts/metre}^2$$

where

c = the velocity of light $= 3 \times 10^8$ metres/sec.

The power loss in the surface $\mathbf{s}$ of the resonator will be

$$P_R = \frac{\pi c\delta\mu_1}{2\lambda}\int H'^2_{tan} . d\mathbf{s} \text{ (watts)} \quad .. \quad (5)$$

We are led to the postulation of a quantity which expresses the energy and power relations in an unloaded cavity. We may utilize the familiar relationship :—

$$Q = \frac{2\pi \text{ (maximum instantaneous energy present in the } \begin{Bmatrix}\text{electric}\\ \text{or}\\ \text{magnetic}\end{Bmatrix} \text{ field)}}{\text{(energy loss in one period)}}$$

(Bib. 3, p. 377.)

Moreover, $\frac{c}{\lambda} \times$ (energy loss per period $= P_R$. and therefore, from equation (2), we have

$$Q = \frac{\omega W_F}{P_R} \quad .. \quad .. \quad .. \quad .. \quad (6)$$

Energy flowing from a modulated electron beam into the electromagnetic field inside

an unloaded resonator will appear as a flow of an equal amount of energy into the walls of the resonator, and (since the rate of flow of energy is equal to power), we have, for the power delivered from the electron beam to the resonator.

$$P_0 = P_R \quad .. \quad .. \quad .. \quad .. \quad (7)$$

The object of a resonator-type valve will be, however, to deliver electromagnetic energy to a load. A part of the electromagnetic field is therefore to be used to transfer some useful power into the load. This transfer may take place either by the direct penetration of a part of the field into a conductor or semi-conductor, or by the radiation of the energy into space. P_R should be as small as possible, because it is merely waste.

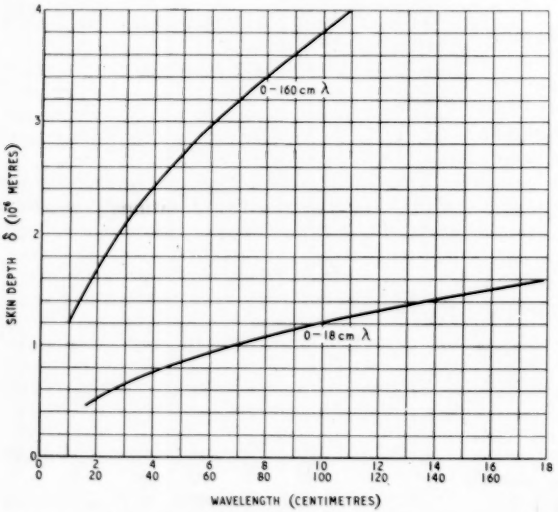


Fig. 1. *The depth of penetration of an electromagnetic wave into a copper surface (skin depth).*

Imagine, then, that a load is coupled to the resonator field (by, for instance, a coupling loop), and, therefore, that,

$$P_0 = P_R + P_L \quad .. \quad .. \quad .. \quad (8)$$

where P_L = the power delivered to the load.

It is to be assumed that no energy escapes from the resonator other than to the load, and that the energy from the electron beam reaches the load and the copper walls of the resonator only through the field.

A further necessary assumption is that the addition of the load, and the existence of the copper losses, do not cause the fields to depart appreciably from the shape assumed when copper losses are negligible and the load does not exist.

We may then write down an expression for the energy and power relations in a loaded resonator which will correspond with that of equation (6) for an unloaded one, namely

$$Q_L = \frac{\omega W_F}{P_R + P_L} \quad .. \quad .. \quad .. \quad (9)$$

It will also be found desirable to establish a quantity which may be looked upon as the efficiency of the resonator and load in combination, namely :—

$$\eta_c = \frac{P_L}{P_L + P_R} \quad .. \quad .. \quad .. \quad (10)$$

Then we have

$$\frac{1}{1 - \eta_c} = \frac{P_L + P_R}{P_R}$$

and it follows that

$$P_L + P_R = \frac{P_R}{1 - \eta_c}$$

Substituting the above in (9), we have

$$Q_L = \frac{\omega W_F}{P_R}(1 - \eta_c)$$

and from (6) we get

$$Q_L = Q(1 - \eta_c) \quad .. \quad (11)$$

3. Energy Transfer from a Modulated Electron Beam to an Electric Field

Turning next to the evaluation of the power P_0 delivered by the beam of electrons into the resonator, it is necessary to establish the conditions determining its magnitude. This involves a study of the energy relationships of electrons injected into an electric field. Whilst the principles of this mechanism are well known, quantitative relationships appear to have been set out only in the sketchiest forms in the literature. Recently, however, T. S. Popham has computed much of the necessary information.* In his work, an electron beam current is assumed to travel in an electric field between two parallel infinite plane electrodes, and has a value

$$I_0 = Ai \quad .. \quad .. \quad .. \quad (12)$$

* See a paper to be published in due course.

where

A = the cross-sectional area of the beam,

and i = the current density of the beam.

The electric field **E** between the electrodes is assumed to be wholly normal to their surfaces, parallel to the entrance directions of the electrons, and to be a linear function of the distance l_1 between the electrodes. Curl **E** = zero, and consequently an alternating potential may be considered to exist between the electrodes which has a maximum instantaneous value

$$V'_{\mathrm{B}} = \int_{l=0}^{l=l_1} E'_l \, dl = E'_1 l_1 \text{ (volts)} \quad .. \quad (13)$$

where

E'_l is the field strength between the electrodes.

The electrons are assumed to enter this space at a steady initial energy V_{B} as defined by equation (1), and, throughout this paper, the velocity v_0 is assumed not to be large enough for relativity effects to be appreciable.

In this electron-beam — electric-field system there will exist a ratio

$$M = \frac{V'_{\mathrm{B}}}{V_{\mathrm{B}}} \quad .. \quad .. \quad .. \quad .. \quad (14)$$

Another parameter is the transit angle ϕ of an electron which is assumed to travel between the electrodes when $M \to 0$. This may be referred to as the d.c.- or "small signal"-transit angle. It is equal to

$$\phi = \frac{10^3 l_1}{\lambda V_{\mathrm{B}}^{\frac{1}{2}}} \pi \text{ (radians)} \quad .. \quad .. \quad (15)$$

We may specify the efficiency of transfer of power from the beam to the resonator-load system by the factor

$$\eta_0 = \frac{P_0}{I_0 V_{\mathrm{B}}} = \frac{P_0}{P_{\mathrm{B}}} \quad .. \quad .. \quad .. \quad (16)$$

where

P_{B} = the d.c. power in the beam.

For any given values of ϕ and M, and of the waveform of the modulation of the beam current, there exists a value of η_0.

Throughout this paper it will be assumed that the resonator is operating at a mode such that it is tuned to the frequency of the fundamental component of the electron-beam modulation. Note that it is possible to provide a resonator that possesses a number of resonant modes which are simultaneously equal to a corresponding number of component frequencies of the electron-beam modulation shape.

In T. S. Popham's analysis, it has been shown that for any given value of the "small signal"- transit angle ϕ there exists an optimum value of $M = M_0$. This relationship, of course, is itself a "large signal" condition (i.e., when $V_{\mathrm{B}} \approx V'_{\mathrm{B}}$), but it is a matter of mathematical and practical convenience to relate it to the "small signal" parameter ϕ [(eqn. (15)] which may be readily evaluated.

Fig. 2 is obtained from T. S. Popham's work, and shows η_0 and M_0 as a function of ϕ for two typical waveforms.

M_0 varies with the beam current waveform at any given value of ϕ; but not very greatly. In what follows it will be assumed that the variation of M_0 with ϕ is that indicated in Fig. 2, and is the same for all beam current waveforms. In practical microwave valves, moreover, the difficulties of obtaining special waveforms are such that there seems no point in specifying in greater detail the comparatively minor effects of waveform upon M_0.

When ϕ is appreciable in magnitude (e.g., under microwave conditions), it is not perhaps obvious that the efficiency will fall if $M > M_0$. In fact, T. S. Popham's analysis is then no longer rigorous. It does not include the "proximity effects" of electrons, and when $M > M_0$ it indicates that different electrons may be in the same place at the same time. This, of course, is absurd; but an analysis which takes into account what really happens—that is, an analysis

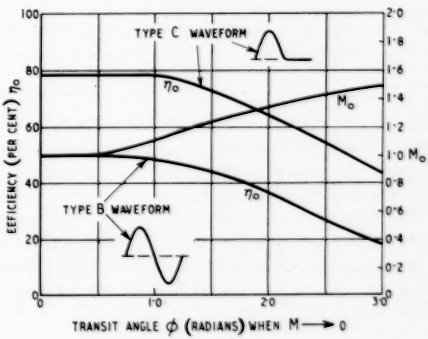


Fig. 2. *The efficiency η_0 of transfer of power from an electron beam into an electric field as a function of the d.c.-transit angle ϕ. The ratio M of the peak voltage to the entrance energy in volts is also shown, (after T. S. Popham).*

allowing for proximity effects—does not yet appear to exist in the art. In the present paper it will be assumed, as a result of approximate computations, that when M exceeds M_0, the efficiency will be reduced.

To apply these results to resonant cavities it will be assumed (as is in fact justified in the resonator shapes to be used) that curl $\mathbf{E} = 0$ along the path of the beam in the resonator, though not necessarily so elsewhere. The line integral of the field along the path of the beam may then be looked upon as a potential difference and measured in volts.

It will be shown later in this paper that the technique of beam production and modulation is such that it is desirable for the field integral V'_R to be as great as possible in a given resonator, and (because E'_1 is assumed to vary directly as l) we have the requirement (from 13) that ϕ should be as great as possible. But ϕ is proportional to l_1, and therefore should be as large as possible also. By reference to Fig. 2, it will be seen that η_0 commences to fall appreciably when ϕ exceeds $\frac{\pi}{2}$. These factors are found to result, on microwaves, in a choice of ϕ such that it must be theoretically in the neighbourhood of an optimum $\phi_0 = \frac{\pi}{2}$.

We have, also, the requirement that M should equal M_0. From Fig. 2, when $\phi = \frac{\pi}{2} = \phi_0$, then $M_0 = 1.24$. These two numerical values will be used for M_0 and ϕ_0 throughout this paper. Neither of these optima are at all critical from the point of view of manufacturing tolerances.

For the purposes of this paper, we may therefore restrict η_0 in equation (16) as expressing the efficiency of transfer of power from the beam to the resonator-load system in the conditions when $\phi = \phi_0$ and $M = M_0$.

4. Power Transfer from a Modulated Electron Beam to a Resonator

The problem with which the present paper deals is the transfer of this power P_0, first from the beam into a resonator, and secondly to a load. It is therefore necessary to establish a relationship between the power conditions of the resonator-load system itself and the electron beam relationships of equations (1), and (12) to (16).

If the integral in equation (13) is along a path of an electron beam which is injected into a resonator, we have a means of transferring power from the electron beam to the electromagnetic field in the resonator. A load may then be coupled to the field and fed with power P_L. The overall efficiency of the whole system will be, from equations (10) and (16),

$$\eta = \eta_0 \eta_c = \frac{P_L}{P_B} \quad .. \quad .. \quad .. \quad (17)$$

In any resonator which is to be set into oscillation at a given mode, there exists theoretically an infinite number of different paths along which the electron beam can travel. It will be assumed that V'_R is substantially the same over a cross-sectional area A_0 of the beam along the path chosen, and, as previously mentioned, that curl $\mathbf{E} = 0$ along l_1. For the reasons already mentioned, the beam path is chosen, out of those available, to be that along which the integral in (13) is at its maximum at the operating mode of the resonator.

We can then specify an important relationship which expresses the properties of the resonator field shape and working mode with respect to the injection of energy by means of an electron beam, namely what may be termed the "voltage coefficient" of the resonator:—

$$\xi = \frac{V_R'^2}{4\pi f W_F} \quad .. \quad .. \quad .. \quad (18)$$

From the foregoing (if the beam efficiency η_0 is to be at its maximum), $M = M_0$, and $\phi = \phi_0$; and therefore, from (14), we get

$$V_B = \frac{V'_R}{M_0} \quad .. \quad .. \quad .. \quad .. \quad (19)$$

For any given resonator and transit angle, we have then, from (15),

$$V_B^{\frac{1}{2}} = \frac{10^3 l_1}{\phi_0 \lambda} \pi$$

l_1/λ will not, of course, vary with λ in a resonator of given shape.

We have then the requirement that the small signal transit angle $\phi_0 = \frac{\pi}{2}$; therefore we have a value of V_B (to be designated by V_{OB}), which does not vary with wavelength, and which is a fixed property of any given resonator shape, mode and position of the beam path through the resonator. It may be called the "characteristic voltage" of the resonator.

Therefore, from (15), we may write

$$\frac{l_1}{\lambda} = a = 5 \times 10^{-4} V_{OB}^{\frac{1}{2}} \quad .. \quad .. \quad (20)$$

and

$$V_{OB} = 4 \times 10^6 a^2 \text{ (volts)} \quad .. \quad (20.1)$$

Equation (20.1) is plotted in Fig. 3.

We are now in a position to link together the beam and resonator equations. We have, from (9),

$$P_R + P_L = \frac{\omega W_F}{Q_L}$$

From (8), and by substituting from equation (11), we have

$$P_0 = P_L + P_R = \frac{\omega W_F}{Q(1 - \eta_c)} \quad .. \quad (21)$$

Substituting from (19) in (18), we have, for the resonator voltage coefficient (since $V_B = V_{OB}$)

$$\xi = \frac{M_0^2 V_{OB}^2}{2\omega W_F} \quad .. \quad .. \quad .. \quad (22)$$

Combining (21) and (22) gives the fundamental electron-beam—resonator-load relationship we require, namely,

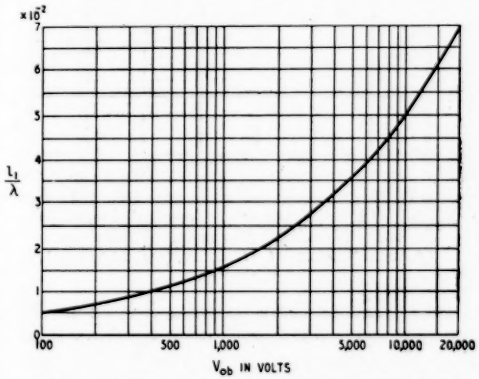


Fig. 3. *The length of the electron-beam path in terms of wavelength as a function of resonator "characteristic voltage."*

$$P_0 = P_R + P_L = \frac{M_0^2 V_{OB}^2}{2\xi Q(1 - \eta_c)} = \frac{1.54\, V_{OB}^2}{2\xi Q\,(1 - \eta_c)} \quad .. \quad .. \quad (23)$$

We have, then, for any given shape of resonator and mode of oscillation, the following parameters :—

The " voltage coefficient " ξ

The " characteristic voltage " V_{OB}.

The " selectivity factor " Q.

It follows that, when equation (23) holds, $M = M_0$, and the small signal transit angle $\phi = \phi_0$, and the efficiency of transfer of power from the beam to the resonator-load system is η_0. The beam path is also chosen so that V'_R is at its maximum.

By substituting for P_0 in (23) from (16), and remembering that $V_B = V_{OB}$, we have

$$V_{OB} I_0 \eta_0 = \frac{1.54\, V^2_{OB}}{2\xi Q(1 - \eta_c)}$$

and

$$\eta_0 = \frac{1.54\, V_{OB}}{2\xi Q I_0 (1 - \eta_c)} \quad .. \quad .. \quad (24)$$

Note that an important factor in this equation is the ratio V_{OB}/I_0, which is plainly a characteristic of the particular beam-forming and modulating devices used, and of the area A_0 of the resonator over which V'_R is sufficiently uniform to be utilized as the path of the beam of electrons. We may then define a parameter, which will be typical of the particular electron-beam producing and modulating device used; namely, a ratio between the electron beam voltage and the current density,

$$\Delta = \frac{V_B}{i} = \frac{V_{OB}}{i} \quad .. \quad .. \quad .. \quad (25)$$

Thus, we have,

$$V_{OB}/I_0 = \frac{V_{OB}}{iA_0} = \frac{\Delta}{A_0} = R_B \quad .. \quad (26)$$

and equation (24) becomes

$$\eta_0 = \frac{1.54\Delta}{2\xi Q A_0 (1 - \eta_c)} \quad .. \quad .. \quad (27)$$

Δ is determined partly by theoretical considerations, which are set out in the Appendix, and in Fig. 4, for " focused "-beam valves, where $V = V_{OB}$; but to a great extent by considerations of valve engineering which, in the case of a given kind of valve (e.g., according to whether it is a klystron or a magnetron) will be affected by definite limitations. These limitations will be referred to again later.

Solving (27) for the resonator—load efficiency, we have

$$n_c = 1 - \frac{0.77\Delta}{\xi Q A_0 \eta_0} \quad .. \quad .. \quad (28.1)$$

or $$\eta_c = 1 - \frac{0.77 R_B}{\xi Q \eta_0} \quad .. \quad .. \quad (28.2)$$

Clearly, η_c should be as nearly unity as possible. Since $\frac{0.77}{\eta_0}$ is a constant for any given beam-current-modulation shape, the important engineering quantity is a ratio which may be referred to as the " resonator-beam ratio," namely,

$$\Psi = \frac{\xi Q}{R_B} \quad .. \quad .. \quad .. \quad .. \quad (29)$$

η_c is plotted as a function of Ψ in Fig. 5.

It will be seen from this that η_c does not approach a reasonable value unless Ψ is at least as large as 10. It should be as great as possible, and from (18) this means that the integral V'_R must also be as great as possible.

R_B is, from (26), in the nature of a ratio between a voltage and a current; but, because it varies with voltage, this ratio must not be looked upon as a "resistance." Similarly, we may define a parameter of major importance, which may be called the "electronic coupling factor" namely

$$\zeta = \xi Q = \left(\frac{V'^2_R}{2\omega W_F}\right)\left(\frac{\omega W_F}{P_R}\right) = \frac{V'^2_R}{2P_R} \quad (30)$$

There is perhaps a temptation to look upon ζ as an "impedance" by analogy with lumped-constant low-frequency valve circuits; but, in fact, the analogy is not valid because its use infers the various special conditions already set out (for example $V'_R = V_{OB}M_0$, $\phi = \phi_0$, and $M = M_0$), and the parameter is not limited by "lumped constant" approximations.

Another useful relationship can be deduced. From (17) we have

$$P_L = V_{OB}I_0\eta_0\eta_c \quad .. \quad .. \quad .. \quad (31)$$

Substituting (28.1) in (31)

$$P_L = I_0V_{OB}\eta_0\left\{1 - \frac{0.77\Delta}{\xi QA_0\eta_0}\right\} \quad .. \quad (32)$$

From (26) this may also be written

$$P_L = \frac{V_{OB}^2\eta_0A_0}{\Delta}\left\{1 - \frac{0.77\Delta}{\xi QA_0\eta_0}\right\} \quad .. \quad (33.1)$$

or, from (26) and (29),

$$P_L = \frac{V_{OB}^2\eta_0}{R_B}\left\{1 - \frac{0.77}{\Psi\eta_0}\right\} \quad .. \quad .. \quad (33.2)$$

5. Notes on the Basic Relationships

Before proceeding further, the following general comments should be noted:—

The requirement that the overall efficiency η should be at a maximum applies to both power oscillators and to those electron-beam valves which may be employed for other purposes—such as to drive other valves. Both the maximum possible power output, and the maximum possible resonator field strength, require that η should be as large as possible.

During operation, changes in the effective value of I_0 may be utilized. The waveform of the modulation may be changed, or its depth, or the average current may be reduced. Then, over the range of $M < M_0$, M will vary with the amplitude of the a.c. component I_{ac} of I_0 to which the resonator is tuned. P_L and η_0 will vary as the square of I_{ac}. As previously-mentioned, when $M > M_0$, a relationship holds which is indeterminate, and, in general, theoretically undesirable.

It will be observed that ξ varies as V'^2_R. If, for any special purpose, it is required to vary ξ in a given resonator, and to reduce

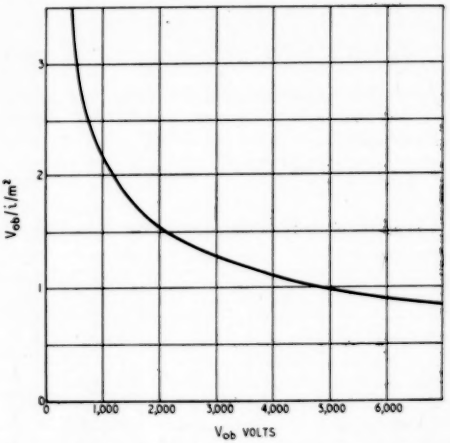


Fig. 4. *The approximate theoretical limits for the ratio of voltage to current density in a "focused" beam of electrons (see Appendix), $V = V_{OB}$.*

it from the maximum possible, it is generally possible to choose a path of integration l_1 so that the reduction is obtained. This procedure must always reduce the resonator-load efficiency η_c.

It will be clear, therefore, that, in order to meet the various requirements of microwave electron-beam technique, many different resonator shapes and sizes will be needed, and these shapes must have "characteristic voltages" which vary over a wide range. The square root of the resonator "characteristic voltage" $V_{OB}^{\frac{1}{2}}$ may well be required to have values which vary from the order of 10 to the order of 140 or more.

There is, theoretically, a limitation to P_L due to the heat loss P_R; but, in fact, with a properly designed system, this loss is generally negligible. In any case the temperature of the resonator is a function of the general mechanical design of the tube as regards cooling, and the heat loss P_R is generally completely swamped by the cooling requirements of the cathode and of the waste

part of the beam power P_B—P_o. It will not be considered further in this analysis.

6. The Integral Forms of Ψ

It is now necessary to examine the parameter Ψ in greater detail, particularly with reference to its dependence upon wavelength.

From equations (29) and (30), we may write for this resonator-beam coefficient

$$\Psi = \frac{\xi Q}{R_B} = \left\{\frac{V_B'^2}{2\omega W_F} \cdot \frac{\omega W_F}{P_B} \cdot \frac{A_0}{\Delta}\right\}$$

$$= \left\{\frac{\left[\int_{l=o}^{l=l_1} E_l' dl\right]^2}{\frac{4\pi c}{\lambda}\frac{\epsilon_0}{2}\int \mathbf{E}^2 d\mathbf{v}}\right\}\left\{\frac{\frac{2\pi c}{\lambda}\cdot\frac{\mu_0}{2}\int \mathbf{H}^2 d\mathbf{v}}{\frac{\pi c\delta\mu_1}{2\lambda}\int H'_{tan} d\mathbf{s}}\right\}\frac{A_0}{\Delta} \quad .. \quad .. \quad .. \quad (34)$$

Since the denominator of ξ is twice the numerator of Q, this reduces to

$$\Psi = \frac{\left[\int_{l=o}^{l=l_1} E'_l \cdot dl\right]^2}{\frac{\pi c\delta\mu_1}{\lambda}\int H'_{tan} d\mathbf{s}}\left(\frac{A_0}{\Delta}\right) = \frac{V'^2_B}{2P_B}\cdot\frac{1}{R_B} \quad .. \quad .. \quad .. \quad (35)$$

For a discussion of the integrals which specify the losses and fields in resonators, see Sarbacher and Edson's text-book, (Bib. 3).

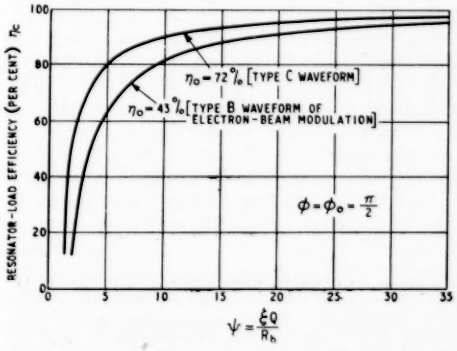


Fig. 5. *The efficiency η_c of transfer of power from a resonator to a load as a function of the ratio between the resonator "electron-coupling coefficient" $\xi Q = \zeta$ and the working voltage/current ratio R_B of the electron stream.*

7. The Effect of Wavelength Variations

It will be found that $\zeta = \xi Q$ varies as $\lambda^{\frac{1}{2}}$. It may be shown also that the effective beam cross-sectional area A_0 usually varies as the square of the wavelength, although it is sometimes possible to utilize over a limited range of wavelengths, as one dimension of A_0, a dimension of the resonator which does not vary with wavelength. With this latter reservation, then, we have

$$A_0 = k\lambda^2 \text{ (metres}^2\text{)} \quad .. \quad .. \quad (36)$$

Then, it follows from this, and from (34), that Ψ varies as $\lambda^{5/2}$ which suggests (as indeed will be shown to be the case later) that the efficiency will fall very sharply indeed when λ becomes less than some quite definite value.

8. Uses of a "Reference Wavelength"

Unfortunately, R_B is not determined exclusively by the theoretical considerations of, for instance, the Appendix, but by the quite arbitrary sizes in which it is practicable to make vacuum apparatus, and, in fact, by the general constructional requirements of vacuum engineering. It is necessary for many reasons to use different techniques at different parts of the wavelength spectrum. There is no general voltage/current function of wavelength to insert in the equations. For this reason a practical difficulty arises in expressing analytically the general performance that may be expected from resonators and beams in combination.

This difficulty will be overcome if we use an idea of splitting up the wavelength spectrum into arbitrary, and if necessary, overlapping bands of wavelengths. Then, for a given resonator used over a given range of wavelengths, and for a given sort of electron-beam modulating and producing device, a value of the beam area A_0 at a given wavelength (to be referred to as the "reference wavelength" λ_0) may readily be specified.

In accordance with this idea, the resonator and beam parameters which vary with wavelength in equations (34), (35), and (36) will all be evaluated at a reference wavelength λ_0, and these special values will be designated by a suffix indicating the wavelength in centimetres. Thus, Q_{100} will indicate the value of Q at a wavelength of 1 metre. Using 0 to represent this suffix, we may re-write equations (28), (29) and (30) as follows, noting that it is not necessary to specify the minimum wavelength of each band, because this will automatically be set by the point at which the fall of Ψ (with $\lambda^{5/2}$) indicates that the particular resonator—beam combination is no longer useful. Thus

$$\Psi = \frac{\zeta_0}{R_{B0}}\left(\frac{\lambda}{\lambda_0}\right)^{5/2} = \Psi_0\left(\frac{\lambda}{\lambda_0}\right)^{5/2} \quad .. \quad (37)$$

and (28.2) and (33.2) may be written respectively as

$$\eta_c = 1 - \frac{0.77}{\eta_0}\frac{1}{\Psi_0}\left(\frac{\lambda_0}{\lambda}\right)^{5/2} \quad .. \quad .. \quad (38)$$

and, from (36),

$$P_L = \frac{V_{OB}^2\eta_0}{R_{BO}}\left(\frac{\lambda}{\lambda_0}\right)^2\left[1 - \frac{0.77}{\eta_0}\frac{1}{\Psi_0}\left(\frac{\lambda_0}{\lambda}\right)^{5/2}\right] \quad .. \quad .. \quad .. \quad (39)$$

The foregoing analysis is the general case for the transfer of energy from an electron stream to a field and then to a load, and includes the more familiar " lumped circuit " concepts as special approximate cases.

(To be continued)

(Bibliography will be included at end of Part III of the article.)

H.F. RESISTANCE AND SELF-CAPACITANCE OF SINGLE-LAYER SOLENOIDS

By R. G. Medhurst, B.Sc.

(Communication from the Staff of the Research Laboratories of The General Electric Company, Limited, Wembley, England.)

(Concluded from page 43 of the February issue.)

9. Self-capacitance of Single-layer Coils.

9.1. The self-capacitance of each coil, including capacitance due to leads, has to be added to the parallel capacitance reading of the twin-T. It is a small correction, usually less than 1 per cent. The original intention was to use Palermo's formula for self-capacitance[14], this being available in abac form[18] and hence readily made use of. However, for the closely-spaced coils, a noticeable variation with frequency started to appear in the calculated values of inductance (which should be consistent to better than ½ per cent), so it was decided that an attempt should be made to find out whether Palermo's formula did in fact agree with experiment, and, if there was a substantial disagreement, whether an empirical formula could be substituted.

What was required was a set of formulae, or, preferably, a set of curves from which the self-capacitance of a particular coil could be quickly and easily read off, say to 20 per cent or better. Since the capacitance of the leads is of the same order of magnitude as that of the coil, it was first necessary to find out whether the lead capacitance could be specified by some quantity which would be additive algebraically to the self-capacitance of the coil.

The simplest hypothesis is that the " live " lead can be treated as an isolated straight vertical wire ; that is to say, that (1) the fact of its being bent, and (2) the proximity of the coil to the upper end have a negligible effect on its capacitance. This we shall show to be correct, to the degree of approximation we require.

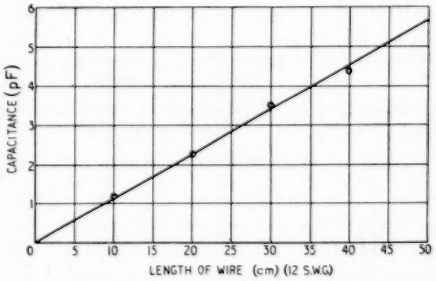


Fig. 4. *Variation with length of capacitance of vertical 12 gauge copper wire.*

9.2. The capacitances of a number of copper wires of various lengths and diameters, from 10 to 40 cm in length and from 12 S.W.G. to 44 S.W.G., were measured at 200 kc/s, the wires standing vertically upright with their lower ends in the live terminal of a Cambridge Capacity Meter. Over this range of length, the capacitances of each wire gauge were quite closely proportional to their lengths (see, for example, Fig. 4).

The capacitances, measured in this way, of 25-cm lengths of wire of various gauges are plotted against the wire diameter in Fig. 5. In Fig. 6 capacitance is plotted against length for a number of wire gauges.*

We may readily show that bending of the wire and alteration of its position relative to earth make no large difference to the measured capacitance. A 25-cm length of No. 12 S.W.G. copper wire was measured in a vertical position, as before. Its capacitance was 2.9 pF. Now, a piece of brass sheet, 35 cm × 15 cm, was attached to the earth terminal of the capacitance meter, so that it formed a horizontal earth

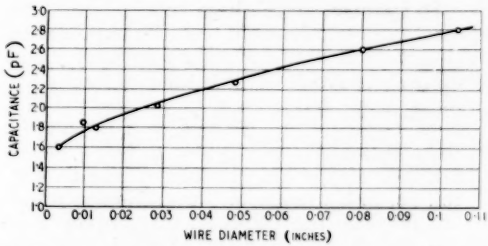


Fig. 5. *Capacitance of 25-cm lengths of vertical copper wire of various diameters.*

adjacent to the wire in the live terminal. The wire was bent over so that about 2/3 of its length was horizontal and about 6 cm above the brass sheet. The capacitance was now 3.0 pF. Even when the wire was brought to within about 2 cm of the earth plate, the capacitance reading only rose to 3.6 pF. Finally, the wire was screwed into the meter terminal at its centre, the two ends being bent up to about 45° to the horizontal. The capacitance reading was now 3.1 pF.

Some of these measurements were repeated on the twin-T, at frequencies up to 20 Mc/s, and close agreement was obtained. In addition, some observations were made on the effect of the proximity of an adjacent vertical earth lead, screwed into the earth terminal of the twin-T. It was found that there was no measurable increase in capacitance until the earth lead was brought to within one or two centimetres of the live lead.

9.3. Before we discuss the effect of the proximity of the coil on the capacitance of the "live" lead, we have to describe the method used for measuring the capacitance of the whole coil-lead assembly.

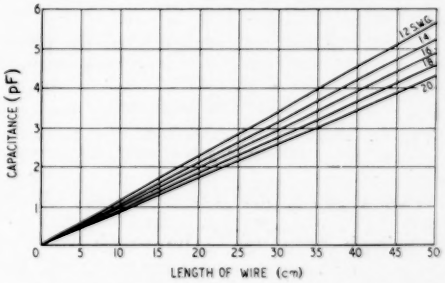


Fig. 6. *Variation of capacitance with wire length for vertical copper wires of various gauges.*

The standard technique for making self-capacitance measurements on coils was originally suggested by G. W. O. Howe[17]. The square of the wavelength is plotted against the added parallel capacitance necessary to resonate the coil. The points so obtained should lie on a straight line, which is produced to meet the capacitance axis, making a negative intercept which is numerically equal to the self-capacitance. The present method is a modification of this, making use of the large range (1,000 pF) of the main tuning capacitor of the twin-T and its fine graduation (0.2 pF per division). A measurement is carried out at the frequency at which the coil resonates with about 1,000 pF. About half a dozen additional measurements are now required, the first at about four times this frequency and the remainder at frequencies increasing in steps of 2 or 3 Mc/s.

Now, if we know the self-capacitance (including lead capacitance), we can calculate the inductance from any one of these measurements, since the coil is resonating with its self-capacitance plus the added capacitance. To obtain a given accuracy of inductance

* It is interesting to note that, over these ranges of length and diameter, the theoretical expression, given originally by G. W. O. Howe (see ref. 19: also ref. 12 p. 116), for capacitance of a straight vertical wire above a plane earth is very roughly linear with respect to the length of wire. Our experimental points, however, fit more closely to a straight line than to this theoretical curve. The theoretical curve for 12-gauge wire intersects the experimental straight line at the 45-cm length point and is about 0.4 pF above at a length of 10-cm. In the 20-gauge case, the theoretical curve falls above the experimental line throughout the range, the maximum deviation being about 0.3 pF.

we need to know the self-capacitance less accurately as the added capacitance becomes higher. In particular, if we make use of the measurement involving an added capacitance of about 1,000 pF, quite a rough value of the

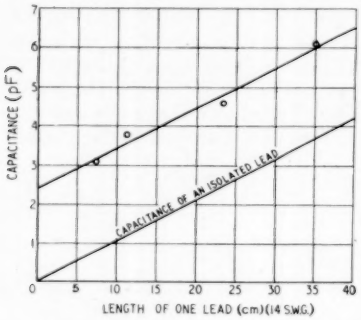


Fig. 7. *Apparent self-capacitance of a coil with various lengths of leads.*

self-capacitance (which is not usually greater than 5 pF) will yield an inductance value of very high accuracy. The rough value is derived from the 1,000-pF measurement and the measurement involving the lowest added capacitance. In practice, we do not actually work out this self-capacitance correction, the inductance being obtained directly from the formula

$$L = \frac{0.02533}{C_2 - C_1}\left[\frac{1}{f_2^2} - \frac{1}{f_1^2}\right]$$

C_1 and C_2 (pF) being the added capacitances at frequencies f_1 and f_2 Mc/s) respectively.

Finally, using this value of inductance we can calculate the self-capacitance, at each of the frequencies of measurement after the first, from the formula

$$C_0 = \frac{0.02533}{Lf^2} - C$$

where C is the added capacitance at frequency f.

As an example of this method, coil No. 32 had 38 turns of 20 S.W.G. copper wire, mean diameter being 5.10 cm, overall length 4.79 cm, spacing ratio 0.720. Self-capacitance measurements took the form shown in Table IV.

The live lead consisted of 10 cm of 14 S.W.G. copper wire. Thus, a lead capacitance of 1.03 pF (independent of frequency) has to be subtracted from each of the readings in Table IV, to give the actual self-capacitance of the coil (see below, Sections 9.4 and 9.6). The mean self-capacitance now becomes 2.30 pF.

It appears, from these results, that the reactance of this coil can be represented closely, over quite a wide frequency range up to and beyond the self-resonant frequency, by a fixed inductance in parallel with a fixed capacitance. This is true for all the coils measured, no evidence being found for the suggestion sometimes made (e.g., ref. 12, p. 84, footnote) that self-capacitance is lower at the self-resonant frequency of the coil than at frequencies much less than this.

9.4 Now we can return to the question of the effect of the lead capacitance on the total measured capacitance.

TABLE IV.

Frequency Mc/s	C_1 (pF)	C_2 (pF)	C	L		C_0
			$C_2 - C_1$ (pF)	Inductance (μH)	$\frac{0.02533}{Lf^2}$ (pF)	$\frac{0.02533}{Lf^2} - C$ (pF)
0.72	100	1076.0	976.0	49.89		
3.0	100	153.2	53.2		56.4	3.2
6.0	200	210.7	10.7		14.1	3.4
8.0	150	154.6	4.6		7.9	3.3
12.0	200	200.15	0.15		3.52	3.4
15.0	300	298.95	−1.05		2.25	3.3
18.0	200	198.2	−1.8		1.57	3.4
					Mean.	3.33

A coil was constructed (39 turns of 20 gauge wire, mean diameter 5.08 cm, overall length 4.70 cm) having 14 gauge leads each 35 cm in length, inclusive of the portion bent over near the twin-T terminals. The parallel capacitance of coil plus leads was measured as just described, and the measurements repeated when the leads were shortened to 23.5, 11.5 and 7.5 cm.

The results are plotted, in Fig. 7, against the length of the live lead. If the lead capacitance adds algebraically, without modification, on to the coil self-capacitance, these points should lie on a straight line

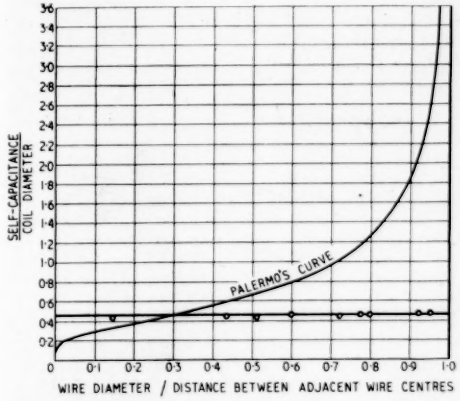


Fig. 8. *Comparison between Palermo's formula and measured self-capacitances of coils having length/diameter = 1 approximately.*

parallel to the 14 S.W.G. line of Fig. 6. By the method of least squares, the best fitting straight line has been drawn among these points, which deviate from it by not more than 5%. This line, it will be seen, is very closely parallel to the 14 S.W.G. line.

9.5. We are now in a position to deal with Palermo's self-capacitance formula. Previous work [14–17,20] has established that the self-capacitance of a single-layer coil (C_0) is directly proportional to the coil diameter. It is also independent of the number of turns, provided this number is not too small. The remaining quantities upon which C_0 might depend are the ratio of coil length to diameter, the wire diameter (d) and the spacing of the turns (s). Investigators before Palermo had assumed that C_0 was independent of d and s. Palermo asserted that C_0 varied with d and s according to the following relation:

$$C_0 = \frac{\pi D}{3.6 \cosh^{-1} s/d}$$

where D is the coil diameter (cm).

This result, independent of the length of the coil, was supposed to hold for coils whose length/diameter ratio was equal to or less than 1.

Fig. 8 shows measured values of the ratio C_0/D for nine coils having diameters ranging from 2.6 to 6.4 cm and spacing ratios (d/s) from 0.15 to 0.95. Wire gauges used range from 18 to 30 S.W.G. All the coils were wound with bare wire on grooved Distrene formers except two, with values of d/s equal to 0.947 and 0.919, which were wound respectively with single-silk-covered and double-silk-covered wire on ungrooved Distrene rod, the turns being as close together as possible. Values of length/diameter were all about 1, ranging from 0.94 to 1.49. Each coil was measured as described above (see Table IV), lead capacitances being subtracted. In Fig. 8, Palermo's theoretical expression for C_0/D is plotted against d/s, and the experimental values are plotted on the same scale. To better than 5%, the measured values fit the expression

$$C_0 = 0.46\ D,$$

being independent of the spacing ratio. These observed values show a tendency to increase slightly with increasing proximity of turns, but this increase was of the order of magnitude of the experimental error anticipated, and it was not thought than any useful conclusions could be drawn.

It has to be pointed out that this experimental demonstration of the lack of dependence of self-capacitance on the spacing of turns contradicts not only Palermo's theory but also some experimental confirmation which he brought forward (see Section 9.7 below.) Consequently, it seems advisable to remark that no investigators other than Palermo have found a measurable variation with turn spacing. J. C. Hubbard [16], for example, says:

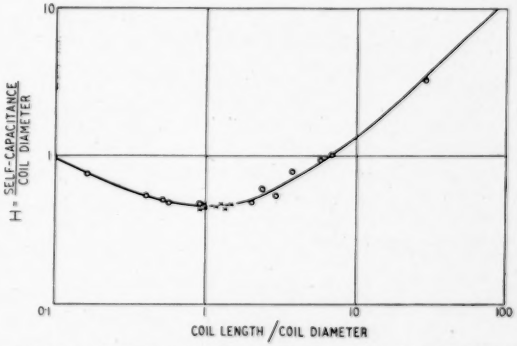


Fig. 9. *Variation of self-capacitance with coil length (one end of coil earthed).*

"There is no evidence that the variation of ratio of pitch to diameter of wire has a measurable effect on the distributed capacity in the region studied, though some effect is to be expected for coils of a smaller number of turns than those studied here." Hubbard's minimum number of turns was 35, and he worked down to a length/diameter of about 0.2; i.e., his coils are "short" enough for Palermo's formula to be applicable.

9.6. C_0 having been shown to be substantially independent of d/s, the final step is to find the variation of C_0 with the length/diameter ratio. Fig. 9 shows the results of a series of measurements on coils whose length/diameter ranged from 29.2 to 0.163. Diameters ranged from 0.675 to 6.36 cm, and numbers of turns from 10 to about 636. All the coils were those which had been used for h.f. resistance measurements, except the two with the greatest and smallest ratios of length/diameter. The former was wound with about 636 turns of 34 gauge wire, double-silk-covered, on a $\frac{1}{4}$-in Distrene former, and the latter with 10 turns of 20 gauge wire, double-silk-covered, on a $2\frac{1}{2}$-in Distrene former.

It appears that, in the commonly occurring case when one end of the coil is at earth potential, we can write down the self-capacitance in the form

$C_0 = HD$ picofarads, where D is in centimetres.

H depends on the length/diameter ratio only. The table of values of H which follows is based on the curve of Fig. 9. The use of these values, with the appropriate lead correction, should give results accurate to 5% or better.

TABLE V.

Length / Diameter	H	Length / Diameter	H	Length / Diameter	H
50	5.8	5.0	0.81	0.70	0.47
40	4.6	4.5	0.77	0.60	0.48
30	3.4	4.0	0.72	0.50	0.50
25	2.9	3.5	0.67	0.45	0.52
20	2.36	3.0	0.61	0.40	0.54
15	1.86	2.5	0.56	0.35	0.57
10	1.32	2.0	0.50	0.30	0.60
9.0	1.22	1.5	0.47	0.25	0.64
8.0	1.12	1.0	0.46	0.20	0.70
7.0	1.01	0.90	0.46	0.15	0.79
6.0	0.92	0.80	0.46	0.10	0.96

J. C. Hubbard[16] remarked: ". . . we apparently have two quite independent factors" (determining the self-capacitance of coils), "one predominating greatly in very short coils, the other, in very long coils." It is an interesting confirmation of this suggestion that the experimental results of Fig. 9 and Table V can be fitted quite closely (to 2 or 3%) by an expression of the form

$$H = 0.1126\,\frac{l}{D} + 0.08 + \frac{0.27}{\sqrt{l/D}}$$

The first numerical factor follows from Nagaoka's inductance formula for long coils and the experimental fact that the self-resonant wavelength for long coils equals twice the length of winding (see below). The other two factors are empirical.

A few additional measurements were made on some two-turn and single-turn coils. A coil of two turns of closely-spaced 18 S.W.G. double-silk-covered wire, diameter 6.47 cm, length/diameter 0.042 gave an H value of 1.53, which is quite close to the value, 1.40, calculated from the expression above. Another two-turn coil, of closely-spaced double-silk-covered 40 gauge wire, diameter 4.46 cm, length/diameter 0.0067, gave the low H value of 0.96. The lead correction is uncertain in both these cases, the assumptions about the live and the earth leads needing modification when the length of the lead becomes comparable with the winding length. It seems from these results that the curve of Fig. 9 can be extrapolated to a length/diameter of about 0.05, even when the number of turns is only two, but that there is a considerable falling off thereafter. A one-turn coil (14 S.W.G., mean diameter 23.9 cm, length/diameter 0.0084) departed even more from the trend of the curve in Fig. 9, the H value being only 0.23.

As an example of the use of Fig. 6 and 9, we may take the coil dealt with in Table III. Ratio of length to diameter was 1.375, and mean diameter was 5.10 cm. Hence, from Fig. 9,

self-capacitance of coil $= 5.10 \times 0.47$
$= 2.4$ pF.

The leads were of 14 S.W.G., the length of each was 9.5 cm. Hence, from Fig. 6,

capacitance of live lead $= 1.0$ pF

Thus, total capacitance $= 2.4 + 1.0$ pF
$= 3.4$ pF.

9.7 The wide discrepancy between

Palermo's results and the present work make it desirable to say something about the theoretical basis of the expression put forward by Palermo.

What is called the "self-capacitance" of a coil will actually be a composite quantity, and the components will not necessarily be mutually dependent. It is convenient, to begin with, to divide coil self-capacitance into two parts, the "internal" and the "external" capacitances. When a current flows through the coil, each turn is at a different mean potential from every other turn. Consequently, there will be capacitances between each pair of turns (modified by the presence of the other turns between or on either side of the particular pair.). We shall call the effective parallel capacitance, across the whole coil inductance, the "internal" capacitance; it is formed by summing all these capacitances between turns, each taken across the appropriate part of the inductance.

Furthermore, each turn will be at a mean potential different from that of the earth, so that each turn will show a capacitance to earth. The effective parallel capacitance formed by summing these capacitances to earth we shall call the "external" capacitance.

It will be apparent that if the external and internal capacitances are comparable in magnitude, the apparent self-capacitance will be different when neither end of the coil is earthed, since the external capacitance will then not appear directly across the terminals of the coil. Hence, the present results, which are all for coils earthed at one end, may not be applicable to coils both ends of which are above earth potential.

Palermo further divides the internal capacitance into two portions, the capacitance between adjacent turns and the capacitance between turns which are not adjacent. He assumes that almost the whole of the self-capacitance is made up of the portion of the internal capacitance between adjacent turns: that is to say, he asserts that the capacitance between non-adjacent turns will be negligible, and he fails to mention the external capacitance.

Now, in spite of having neglected what may be a large part of the total self-capacitance, he predicts values which, for closely spaced coils, are very much larger than the values we have measured. The reason for this over-estimate is not too difficult to see.

Palermo derives his capacitance between adjacent turns from the formula for the capacitance between long parallel cylinders, diameter d and separation of centres s, which he quotes in the form

$$C = \frac{1}{3.6 \cosh^{-1} s/d} \text{ picofarads/cm.}$$

When s/d approaches 1, that is to say, when the cylinders are very close, this expression approaches infinity. However, when the turns of a coil are very close the self-capacitance does not approach infinity, and the reason for the discrepancy appears to be that what we have to concern ourselves with is the effective current-carrying path and not the whole of the cross section of each turn.

When high-frequency current flows through an isolated wire, the current tends to be concentrated near the surface. When the wire is bent into the form of a coil, the current tends, further, to flow round the inner surface of the coil. Finally, the effect of the adjacent turns is to cause the current to withdraw from the portions of the wire nearest to these turns. Thus, even when the turns are very close the effective current-carrying paths are still comparatively remote from each other.

Thus, the capacitance between adjacent turns will be less than that predicted by Palermo. The fact that self-capacitance is substantially independent of spacing of turns suggests that the part of the self-capacitance considered by Palermo is actually negligible.

The question of the validity, or otherwise, of Palermo's formula is complicated by the existence of some measurements (on coils earthed at one end) which he brings forward in support of his theory. It is difficult to say much about these measurements, except that they are closely in agreement with Palermo's formula, and consequently, when the turns are closely spaced, they are very different from other published results on similar coils. The discrepancy is drastically illustrated by Palermo's coil No. 9, which had a diameter of 10.40 cm and a length of 9.65. The number of turns was 28, the wire diameter 0.326 cm and the spacing ratio 0.94. The coil was measured at a "high frequency"; i.e., at something below, but of the order of magnitude of the self-resonant frequency. From the curve of Fig. 9 we would predict a self-capacitance of

4.8 pF. Palermo's formula gives 20.5 pF. The measured value he gives as 20.0 pF.

Palermo's measured coils fall into two groups. Seven of them, with spacing ratios between 0.3 and 0.8 were measured by the Bureau of Standards. Over this region of spacing ratio, Palermo's " proximity effect " is not too pronounced. The measured values were all between 1 and 3 picofarads larger than the values that would be predicted from our present work. Palermo makes no mention of a correction for leads and terminals, and possibly this accounts for the discrepancy. The remaining twelve coils were measured by Palermo himself, and it is among these that we find the capacitances (such as the one already quoted) which are so greatly different in magnitude from our results.

9.8. We have seen that, so far as self-capacitance is concerned, a single-layer coil behaves very closely like a cylindrical current sheet. It is well known that this is also true of the inductive part of its reactance. If we combine these two current-sheet formulae we might expect to deduce some simple expression, depending on the coil geometry, for the self-resonant frequency.

Our measurements have given a self-capacitance expression in the form

$C_0 = HD$, where H is a quantity dependent on the length/diameter only.

The Nagaoka expression for the inductance, L_0, may be written in the form

$L_0 = Kn^2D$, where K is dependent on the length/diameter only.

Now, if we call λ (cm) the self-resonant wavelength, we have

$\lambda = 2\pi c \sqrt{L_0C_0}$ where c (cm/sec) is the velocity of electro-magnetic radiation,

$= 2\pi c \sqrt{HKn^2D^2}$

$= 2\pi c\, nD \sqrt{HK}$

$= Nl$ where N is dependent on the length/diameter only and l is the total length of wire.

Values of N, worked out from the inductances and self-capacitances of the coils previously measured, are plotted against length/diameter in Fig. 10. Table VI gives values of N and has been worked out from Table V and Nagaoka's values of K.

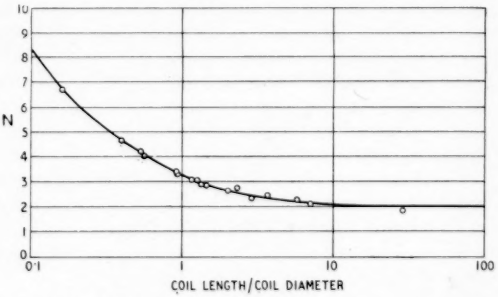


Fig. 10. *Wavelength at self-resonant frequency equals $N \times$ total length of wire.*

TABLE VI.

Length / Diameter	N	Length / Diameter	N	Length / Diameter	N
50	2.0	5.0	2.3	0.70	3.8
40	2.0	4.5	2.4	0.60	4.0
30	2.0	4.0	2.4	0.50	4.3
25	2.0	3.5	2.5	0.45	4.5
20	2.0	3.0	2.5	0.40	4.8
15	2.1	2.5	2.6	0.35	5.0
10	2.1	2.0	2.7	0.30	5.4
9.0	2.1	1.5	2.9	0.25	5.8
8.0	2.2	1.0	3.4	0.20	6.3
7.0	2.2	0.90	3.5	0.15	7.1
6.0	2.3	0.80	3.6	0.10	8.3

10. Frequency Correction.

The measured values of ϕ (ratio of the h.f. resistance of the coil to the resistance of the straightened wire at the same frequency) are mostly for frequencies such that z (see list of symbols) has values between 8 and 20. Though these frequencies are to be regarded as " high " according to our previous definition of " high frequency," (i.e., frequency for which $z>7$), ϕ will still, to some small extent, be frequency dependent. So that the measured values shall be comparable among themselves, it will be advantageous to apply a frequency correction such that the corrected ϕs correspond to the same value of z. If we choose infinity as this standard z value the corrected ϕs can be compared directly with Butterworth's "high-frequency" table, which is supposed to apply at infinitely large z. Since, as we shall see, the frequency correction is small, we may still use the corrected ϕ values at the orders of frequency commonly encountered.

It is unfortunate that exact measurements on coil resistance are almost as scarce at

low as at high frequencies. Consequently, it has not been found possible to deduce from previous work an experimental frequency correction to convert the present measurements from "high" to "infinite" frequency. Tentatively, a correction formula was used based on Butterworth's theoretical considerations, modified in the light of the present results. The formula in question is

$$\Delta\phi = \frac{1}{8G}(\phi_{exp} - 2\alpha)$$

ϕ_{exp} being the measured value of ϕ, and G and α being quantities due to Butterworth (see, e.g., ref. 12, pp. 78 and 79).

The correction did not usually exceed 2%. It may be either positive or negative. In deriving $\Delta\phi$, the general form of Butterworth's resistance formula is assumed; i.e.,

$$\frac{\text{a.c. resistance}}{\text{d.c. resistance}} = \alpha H + kG$$

where the first term represents the losses due to the currents in the wires, and the second the losses due to the field of the whole coil. H and G are functions of z only, being given for large z by $\frac{\sqrt{2}z + 1}{4}$ and $\frac{\sqrt{2}z - 1}{8}$ respectively (the value of z chosen for each coil being that corresponding to the mean working frequency). α depends on the spacing ratio of the turns, and k on the spacing ratio and the dimensions of the coil.

Now, we have seen previously (Section 3.2) that the Butterworth theory is most open to suspicion in that part of it which deals with losses due to the "mean transverse field." The effect of these losses, in the theory, is to cause k, at infinitely high frequency, to have very high values, especially for close spacing. This is the effect that is not confirmed by the present measurements. So, to derive a frequency-correction formula, we shall assume that k has some value which does not vary with frequency (z being sufficiently high) and, eliminate k between the expressions for ϕ at the frequency of measurement and at infinite frequency. α we may take, according to the theory, as being also very nearly invariable with frequency.

When z approaches infinity, we have

$$\phi = \alpha + \frac{k}{2}$$

$$\text{Also, } \phi_{exp} = \frac{\text{a.c. resistance}}{\text{d.c. resistance}} \cdot \frac{1}{\sqrt{2}z/4}$$

$$= 4\frac{\alpha H + kG}{(8G + 1)}$$

and hence, eliminating k from the expressions for ϕ and ϕ_{exp} and using the relation $2H = 1 + 4G$, we obtain the required expression for ϕ; i.e.,

$$\phi = \phi_{exp} + \frac{1}{8G}(\phi_{exp} - 2\alpha)$$

We shall see later that the argument for assuming k to be substantially independent of frequency, when z is high enough, is not complete, because we have only given reasons for rejecting that part of Butterworth's theory which applies to *high*-frequency coil resistance. We shall consider the low-frequency case in Section 14.

11. Effect of the Proximity of the Twin-T Top.

It was thought that an additional correction might be necessary for losses due to the proximity of the metal top of the twin-T. To ascertain the order of magnitude of this effect, a coil (48 turns of 20 gauge d.s.c. wire, mean diameter 2.70 cm, length/diameter 1.82, d/s 0.89) was measured a number of times, the leads being progressively shortened until the distance of the coil from the twin-T terminals was about its own diameter. The coil was then about 2 diameters above the twin-T top.

There was no significant variation in the

TABLE VII.

Length of each lead cm	Total Resistance ohms	Temperature °C	Total Resistance at 20° C ohms	Resistance of leads (20° C) ohms	Resistance of coil (20° C) ohms
40	$1219 \times 10^{-6}\sqrt{f}$	24	$1210 \times 10^{-6}\sqrt{f}$	$25 \times 10^{-6}\sqrt{f}$	$1185 \times 10^{-6}\sqrt{f}$
15.5	1204 ,,	24	1195 ,,	10 ,,	1185 ,,
8	1212 ,,	25	1200 ,,	5 ,,	1195 ,,
8	1218 ,,	26.5	1203 ,,	5 ,,	1198 ,,
4.5	1211 ,,	27	1195 ,,	3 ,,	1192 ,,

measured resistances, their spread being about 1 per cent. The results are given in Table VII.

The two 8-cm measurements were carried out on successive days. The length of each lead includes the right-angle bend at the twin-T terminal, so that in the case of the last measurement the coil was about 2.5 to 3 cm above the terminal.

12. Results of Measurements.

After all these corrections have been applied, we are left with a set of experimental values of ϕ for various non-integral values of coil length/diameter and d/s. These have to be reduced to a table with the same intervals as those of Table I.

We are assisted in this process by remembering that the error in Butterworth's values has been assumed to be due to excessive weight being given to the transverse field losses. If we work out Butterworth's formula again neglecting the transverse field we obtain another table whose entries are all less than those in the Butterworth table, except for the column corresponding to infinite length/diameter. In this case, the transverse field has disappeared.

Our experimental values all lie between these two sets of values. Consequently, we shall take the case where these two sets of values are equal, i.e. the extreme right-hand column, as the limiting case of our empirical table. This is convenient, because it is not possible to measure coils whose length/diameter is infinite.

Further, measurements on a few coils whose length/diameter ratio was about 8, showed that the experimental values of ϕ were within 1 or 2 per cent. of the Butterworth values when the transverse field is neglected. Since transverse-field effects are less appreciable as the length/diameter ratio increases, we may use this result to fill in the 8 and 10 length/diameter columns.

Measurements on several coils having values of $d/s = 0.2$ and 0.3, with length/diameter ranging from 0.5 to 4, showed that Butterworth's values for these two rows are confirmed by the experimental results. This was used to fill in the three lowest rows, it being assumed that the bottom row, the values in which are close to those for a straight wire, could safely be taken as following Butterworth.

It may be pointed out that this agreement with Butterworth's figures, over the region in which Butterworth's theory might be expected to hold, constitutes indirect evidence of the reliability of the measurements.

There remains the most important portion of the table, that is, the top left-hand quadrant. In general, due to difficulties in accurate grooving, the spacing ratios were not exact multiples of 0.1. However, the spacing ratios of the coils which had been constructed to have $d/s = 0.6$, turned out to be very close to the value aimed at. A smooth curve could thus be drawn through their ϕ values, giving the sixth row from the bottom. By extrapolating the values of coils having d/s about 0.5 and 0.7, using this $d/s = 0.6$ row and then drawing smooth curves, the adjacent rows were obtained, and similarly for the rest of the table.

The final result is Table VIII. The values for $d/s = 1$ are obtained by extrapolation. So are the values for the two left-hand columns.

In general, the experimental points deviate

TABLE VIII.

d/s	Coil Length/Coil Diameter											
	0	0.2	0.4	0.6	0.8	1.0	2	4	6	8	10	∞
1.0	5.31	5.45	5.65	5.80	5.80	5.55	4.10	3.54	3.31	3.20	3.23	3.41
0.9	3.73	3.84	3.99	4.11	4.17	4.10	3.36	3.05	2.92	2.90	2.93	3.11
0.8	2.74	2.83	2.97	3.10	3.20	3.17	2.74	2.60	2.60	2.62	2.65	2.81
0.7	2.12	2.20	2.28	2.38	2.44	2.47	2.32	2.27	2.29	2.34	2.37	2.51
0.6	1.74	1.77	1.83	1.89	1.92	1.94	1.98	2.01	2.03	2.08	2.10	2.22
0.5	1.44	1.48	1.54	1.60	1.64	1.67	1.74	1.78	1.80	1.81	1.83	1.93
0.4	1.26	1.29	1.33	1.38	1.42	1.45	1.50	1.54	1.56	1.57	1.58	1.65
0.3	1.16	1.19	1.21	1.22	1.23	1.24	1.28	1.32	1.34	1.34	1.35	1.40
0.2	1.07	1.08	1.08	1.10	1.10	1.10	1.13	1.15	1.16	1.16	1.17	1.19
0.1	1.02	1.02	1.03	1.03	1.03	1.03	1.04	1.04	1.04	1.04	1.04	1.05

Experimental values of the ratio of the high-frequency coil resistance to the resistance at the same frequency of the same length of straight wire.

from the smoothed curves by 1 or 2 per cent. In three cases the deviation is as high as 3 per cent.

It has already been pointed out (Section 5) that, from physical considerations, it becomes increasingly difficult to construct coils fulfilling the various criteria of Butterworth's h.f. resistance table as one approaches the extreme left-hand side of the table. The difficulty becomes acute in the bottom left-hand quadrant. To cover this region, it was necessary to sacrifice the condition that z should be high. Thus, coil No. 41, which had a mean diameter of 6.24 cm, length/diameter of 0.542, and d/s of 0.256, had to be wound with 30 gauge wire, and z was only 2.79. The correction for frequency was now about 17 per cent. This, however, is not too alarming because in this region Butterworth's formulae predict values of α and k (in the Butterworth expression for ϕ, given above) which are almost independent of frequency for large z.

The entries of Table VIII are shown graphically in Fig. 11. For closely spaced wires, there is a critical value when the length/diameter ratio is about 1. This may have some connection with the parallel phenomenon observable in the case of the self-capacitance (see Fig. 9).

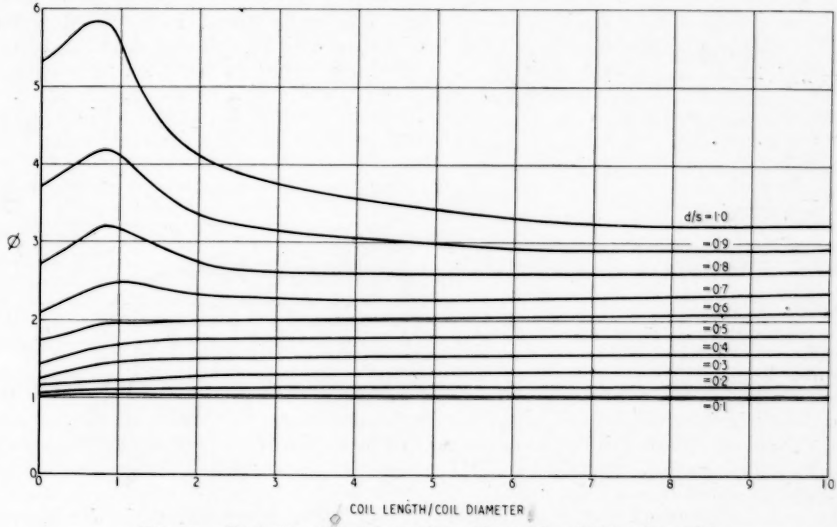


Fig. 11. *Variation of ϕ with spacing ratio and length/diameter ratio.*

$$\phi = \frac{\textit{h.f. resistance of coil}}{\textit{h.f. resistance of same length of straight wire at same frequency}}$$

13. Variation of Q with Coil Shape.

It is not very easy to judge coil performance from figures connected with the h.f. resistance. Normally, we are concerned with coil efficiency, which may best be defined by its Q value at a particular frequency.

It is well-known that Nagaoka's inductance formula may be used, with an error of not more than 5%, up to quite high frequencies. In fact, in the case of the coils used in the present series of measurements, if we assume a constant self-capacitance the inductive part of the reactance agrees closely with Nagaoka's value up to the self-resonant frequency. It breaks down most seriously when the wire diameter becomes comparable (of the order of 1/10th or more) with the coil diameter.

Nagaoka's inductance formula is usually written in the form

$$L_x = \frac{4\pi^2 R^2 n^2 K\, 10^{-9}}{l} \text{ henrys}$$

where R, l and n have the meanings previously defined, and K is a factor involving the ratio of length/diameter only.

Also, R_x = (d.c. resist.). H . ϕ ohms. where H has its high-frequency value (see Section 10) and ϕ is defined by Table VIII.

Hence,

$$R_x = \frac{2\pi Rn}{\pi(d/2)^2}\,\rho \cdot \frac{1}{2\sqrt{2}}\,\frac{\pi d}{4}\sqrt{\frac{2f}{10^9\rho}} \cdot \phi \text{ ohms.}$$

$$= \frac{\sqrt{2}\,Rn\,\rho}{\beta d}\,\phi \text{ ohms,}$$

where

$$\beta = \frac{1}{2\sqrt{2}\pi}\sqrt{\frac{10^9\rho}{f}}$$

Now,

$$Q = \frac{2\pi f\,L_x}{R_x}$$

$$= 2\pi f \cdot \frac{4\pi^2 R^2 n^2}{l}\,K10^{-9} \cdot \frac{\beta d}{Rn\rho\,\phi\,\sqrt{2}}$$

$$= \frac{\pi R}{\sqrt{2}\beta} \cdot \frac{nd}{l} \cdot \frac{K}{\phi}$$

$$= \frac{R}{\sqrt{2}\,\beta}\,\frac{\pi d}{s} \cdot \frac{K}{\phi}$$

$$= \frac{R}{\sqrt{2}\beta}\,\psi \text{ where } \psi \text{ is a function of } d/s \text{ and } l/D.$$

For copper, taking $\rho = 1.7 \times 10^{-6}$ ohm-cm we find that

$$Q = 0.15R\,\psi\,\sqrt{f}$$

Table IX, which gives values of ψ for various values of coil length/diameter and spacing ratio, is derived from Table VIII and Nagaoka's table of K. The measured values of Q (uncorrected for leads) were checked against those predicted from this table. For coils falling within the body of the table, that is to say, to the left of the column length/diameter = 4, the difference was 5 per cent. or less, the values based on Table IX being usually higher than the experimental values. For coils with length/diameter = 5 or more, and d/s greater than 0.5, the measured values tended to be 10 per cent or more lower than the predicted values.

The discrepancy in the case of the long coils is due not to divergence of the measured resistances from the values corresponding to Table VIII but to inductance values different from those predicted by Nagaoka's formula. These coils all had small diameters, in order that a sufficiently large length/diameter ratio could be attained without excessive bulk of coil, and the wire diameter could no longer be regarded as small compared with the coil diameter. Now, Nagaoka's formula is a current-sheet formula and assumes that the thickness of this sheet is negligible compared with the diameter. In using Nagaoka's formula, we have taken the mean diameter of our coil as the diameter of his equivalent current-sheet. However, the current in a coil, at high frequencies, tends to flow round the inner surface, so that the equivalent current-sheet should

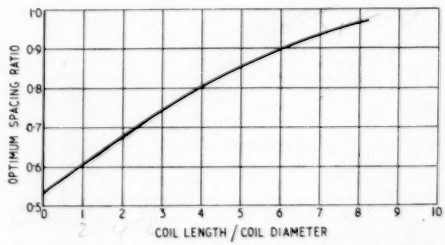


Fig. 12. *Variation of optimum spacing ratio with length/diameter.*

TABLE IX.

d/s	Coil Length/Coil Diameter											
	0	0.2	0.4	0.6	0.8	1.0	2	4	6	8	10	∞
1.0	0.00	0.18	0.26	0.31	0.35	0.38	0.63	0.80	0.89	0.93	0.93	0.92
0.9	0.00	0.24	0.33	0.39	0.43	0.47	0.69	0.84	0.90	0.93	0.93	0.91
0.8	0.00	0.28	0.40	0.46	0.50	0.55	0.75	0.87	0.90	0.91	0.91	0.89
0.7	0.00	0.32	0.46	0.53	0.58	0.61	0.78	0.87	0.90	0.89	0.89	0.87
0.6	0.00	0.34	0.49	0.57	0.63	0.67	0.78	0.85	0.87	0.86	0.86	0.85
0.5	0.00	0.34	0.48	0.56	0.61	0.65	0.74	0.80	0.81	0.82	0.82	0.81
0.4	0.00	0.31	0.45	0.52	0.56	0.60	0.69	0.74	0.75	0.76	0.76	0.76
0.3	0.00	0.25	0.37	0.44	0.49	0.52	0.60	0.64	0.66	0.67	0.67	0.68
0.2	0.00	0.19	0.27	0.33	0.36	0.39	0.45	0.49	0.51	0.51	0.51	0.53
0.1	0.00	0.10	0.14	0.17	0.19	0.21	0.25	0.27	0.28	0.29	0.29	0.30

Values of ψ, from Table VIII and Nagaoka's inductance formula. High-frequency Q of a coil of copper wire or thick tubing is given by $Q = 0.15R\psi\sqrt{f}$.

have a diameter less than the mean diameter of the coil and greater than the inner diameter. That is to say, the measured inductance value should lie between the Nagaoka value obtained by using the mean diameter and that obtained by using the inner diameter. This is found to be the case.

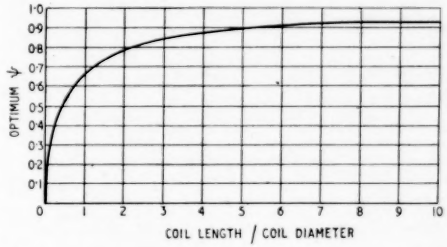


Fig. 13. *Variation of optimum ψ with length/diameter.*

Thus, for coil No. 54 (mean diameter 1.72 cm, wire diameter 0.12 cm), the two Nagaoka values corresponding to the inner and mean diameters respectively were 7.8 and 9.0 μH. The measured value was 8.15 μH., and, subtracting a (calculated) lead inductance of 0.13 μH., the coil inductance was 8.0 μH.

The entries in Table IX increase steadily with increasing length/diameter, except that when length/diameter approaches infinity, and $d/s > 0.4$, there is a small decline. This decline seems not to be readily explainable. The last three columns of the ϕ table, it will be remembered, are those resulting from Butterworth's theory when the transverse field term is neglected. The slight anomaly in the ψ table doubtless means that the effect of the transverse field is not quite negligible for length/diameter ratios of 8 and 10, when d/s is greater than 0.4.

The zero Q values for coils of zero length do not mean that the resistance is infinite, but that the inductance has disappeared.

For a given length/diameter, these entries show a rather flat optimum as the spacing ratio varies. In Fig. 12 the optimum spacing ratio is plotted against length/diameter. In Fig. 13 the value of ψ corresponding to the optimum spacing ratio is likewise plotted against length/diameter.

There is an interesting interpretation of ψ analogous to the interpretation of K in Nagaoka's formula. K may be defined as the ratio of the coil inductance to the inductance of an infinitely long cylindrical current sheet having a diameter equal to the mean diameter of the coil. K, in fact, is an end correction. Similarly, it can be shown from the results in reference 13 that ψ is the ratio of the coil Q to the Q at the same frequency of a certain idealized coil. This "coil" is an infinitely long cylinder, having its inner diameter equal to the mean diameter of the coil we are considering and a wall thickness large compared with the current penetration depth, the current being assumed to flow round the inner surface.

14. Low-frequency Resistance of Single-layer Coils.

When we derived a frequency correction to the measured h.f. resistance values, we assumed that the factor we have called k (in the version of Butterworth's formula given in Section 10) was independent of frequency if z was sufficiently high (of the order of 10 or more). This, it was pointed out, is not even approximately true in Butterworth's theory.

Another way of putting this is that, with close spacing of turns, in Butterworth's theory the h.f. resistance does not become proportional to the square root of the frequency until z is very high. When the turns are touching (physically, but not electrically), the h.f. resistance, in the theory, never becomes proportional to $\sqrt{f}$.

Butterworth gives values for his various quantities for z values up to 5, and for infinite z. Table I is based on these latter values. Interpolation for z values between 5 and ∞ is most conveniently done by plotting Butterworth's functions against the reciprocal of z. The values so obtained are, as one might expect, in closer agreement with the experimental results than those of Table I. Thus, when $z = 10$, for length/diameter = 1 we have the results of Table X.

TABLE X.

d/s	ϕ	% excess over experimental values
1.0	10.37	87%
0.9	5.57	36%
0.8	3.61	14%
0.7	2.61	6%

In the case of the single coil with $d/s = 0.95$, the theoretical value thus obtained

was about 60 per cent. in excess of the measured value.

Thus, if Butterworth's low-frequency values can be relied upon, his predicted resistances are not so wildly in disagreement with experimental results as appears by comparison of Tables I and VIII, especially since in the top left-hand region of the Table, where the discrepancy will be largest, the coils, for physical reasons, had to be constructed with low z values, between 8 and 10.

We can easily show that these low-frequency Butterworth results are open to considerable suspicion. In fact, we shall see that, in consideration of the degree of approximation tolerated by Butterworth, any agreement with observation must, for closely spaced coils, be regarded as in the nature of an accident.

It was stated in Section 3 that Butterworth worked out each of his three types of loss by solving a set of an infinite number of linear equations, each containing an infinite number of unknowns. He used a method of successive approximations.

Now, when the frequency is low, that is to say, for $z = 5$ or less, the amount of arithmetic involved in proceeding beyond the first approximation becomes very large indeed. Consequently, for these values of z, Butterworth uses the first approximation only.

One can only guess at the error this introduces. For the case of touching wires, when z is infinite, there is an infinite error involved if we take only the first approximation for the transverse field losses. That is to say, the entries in the first row of Table I would be decreased from infinity to a series of not too large finite values.

Whether the converse is true, that is, whether if one proceeded to a sufficiently large number of approximations for the case of touching wires at low frequency an infinite result would be obtained, must be a matter of conjecture. On physical grounds, if a coplanar system of an infinite number of infinitely long touching wires offers infinite impedance to a transverse field of very high frequency, it seems not unreasonable to suppose that it will also offer an infinite impedance to a low-frequency transverse field.

Consequently, for different reasons, the applicability of the Butterworth low-frequency formula to specific coils is open to as much doubt as that of his high-frequency formula.

REFERENCES

[1] S. Butterworth. "Eddy-Current Losses in Cylindrical Conductors, with Special Applications to the Alternating Current Resistances of Short Coils." *Phil. Trans. Roy. Soc.*, 1922, A222, p. 57.

[2] S. Butterworth. "Note on the Alternating Current Resistance of Single Layer Coils." *Phys. Rev.*, 1924, Vol. 23, p. 752.

[3] S. Butterworth. "On the Alternating Current Resistance of Solenoidal Coils." *Proc. Roy. Soc.*, 1925, A107, p. 693.

[4] S. Butterworth. "Effective Resistance of Inductance Coils at Radio Frequencies." *Exp. Wireless & Wireless Engineer*, 1926, Vol. 3, pp. 203, 302, 417 and 483.

[5] S. Butterworth. "Designing Low-Loss Receiving Coils." *Wireless World*, 1926, Vol. 19, pp. 754 and 811.

[6] G. W. O. Howe. "The High-Frequency Resistance of Wires and Coils." *J.I.E.E.*, 1920, Vol. 58, p. 152.

[7] C. N. Hickman. "Alternating-current Resistance and Inductance of Single-layer Coils." Scientific Papers of the Bureau of Standards 1923, Vol. 19, p. 73.

[8] W. Jackson. "Measurements of the High-Frequency Resistance of Single-layer Solenoids." *J.I.E.E.*, 1937, Vol. 80, p. 844.

[9] A. H. Meyerson. "Coil Q Factors at V.H.F." *Communications*, May 1944, p. 36.

[10] W. N. Tuttle. "Bridged-T and Parallel-T Null Circuits for Measurements at Radio Frequencies." *Proc. I.R.E.*, 1940, Vol. 28, p. 23.

[11] D. B. Sinclair. "The Twin-T, A New Type of Null Instrument for Measuring Impedance at Frequencies up to 30 Mc." *Proc. I.R.E.* 1940, Vol. 28, p. 310.

[12] F. E. Terman. "Radio Engineers' Handbook." McGraw-Hill Book Company, 1943.

[13] C. R. Burch & N. R. Davis. "On the Quantitative Theory of Induction Heating." *Phil. Mag.*, 1926, Vol. 1, p. 768.

[14] A. J. Palermo. "Distributed Capacity of Single-layer Coils." *Proc. I.R.E.*, 1934, Vol. 22, p. 897.

[15] J. H. Morecroft. "Resistance and Capacity of Coils at Radio Frequencies." *Proc. I.R.E.*, 1922, Vol. 10, p. 261.

[16] J. C. Hubbard. "On the Effect of Distributed Capacity in Single-layer Solenoids." *Physical Review*, 1917, Vol. 9, p. 529.

[17] G. W. O. Howe. "Calibration of Wave Meters." *Proc. Phys. Soc.*, 1912, Vol. 24, p. 251.

[18] P. H. Massaut. "Distributed Capacitance Chart." *Electronics*, March 1938, Vol. 11, p. 31.

[19] G. W. O. Howe. "On the Capacity of Radio-Telegraphic Antennae." *Electrician*, 1914, Vol. 73, pp. 829, 859, 906.

[20] G. W. O. Howe. "Inaugural Address to the Wireless Section." *J.I.E.E.*, 1922, Vol. 60, p. 67.

Radiocommunication Convention

The Institution of Electrical Engineers is holding a convention, covering the wartime activities in the field of radiocommunications, from 25th to 28th March. The convention will be opened by the President of the Board of Trade, Sir Stafford Cripps, at 5.30 p.m. on Tuesday, 25th March, and he will introduce an address by Colonel Sir Stanley Angwin, on "Telecommunications in War."

On the following days there are to be morning, afternoon and evening sessions at which papers covering naval, military, short and long distance, and pulse communications will be read. Propagation, radio components and future trends will also be covered in the convention.

At a further meeting at 5.30 p.m. on 2nd April there will be a paper on C. W. Navigational Aids.

Physical Society's Exhibition

The 31st Exhibition of Scientific Instruments and Apparatus is being held by the Physical Society on 9th–12th April in the Physics and Chemistry Departments of Imperial College, South Kensington, London, S.W.7.

Admission is by ticket only and is restricted to members of the Society from 10 a.m. to 1 p.m., but it is open to non-members from 2 p.m. to 9 p.m.

CORRESPONDENCE

Letters of technical interest are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Television in France

To the Editor, "Wireless Engineer"

MONSIEUR,—Je viens de lire, dans "Abstracts and References," *Wireless Engineer*, Janvier 1947, une analyse (270) sur la Télévision en France.

Un passage m'a particulièrement frappé et mérite une rectification. Votre revue indique en effet que: "La Radiodiffusion Française a un grand studio de télévision construit sur l'ordre des Allemands, dans lequel tout l'equipement est allemand et construit par A. G. Fernseh."

Sur le premier point, je dois vous indiquer que les travaux ont été effectués sur nos plans, et évidemment sous le contrôle allemand; des constructions de cette importance ne pouvaient être entreprises, comme vous devez le savoir, sans une telle surveillance dans un pays occupé.

D'autre part, l'équipement de tout ordre, technique ou artistique, a été construit uniquement et totalement par les constructeurs français. (ainsi, le matériel technique provient de la Compagnie Française de Télévision.

L'erreur qui s'est glissée dans l'article dans *The Journal of the Television Society** est dûe sans doute au fait que les Allemands, pendant leur occupation, ont utilisé leur matériel, qu'ils ont déménagé au moment de leur retraite.

J'espère que vous voudrez bien faire dans votre revue les rectifications nécessaires afin que vos lecteurs aient une vue exacte des efforts réalisés, malgré les difficultés, par l'Industrie française de la Télévision.

Veuillez agréer, Monsieur, l'assurance de ma considération distinguée.

A. ORY,
Le Chef du Service de la Télévision.
Radiodiffusion Française, Paris.

Transient Response of Filters

To the Editor "Wireless Engineer"

SIR,—In the January issue, p. 27, E. T. Emms states that when a voltage $\cos \omega_0 t$ is suddenly applied to a band-pass-filter of any bandwidth, the envelope of the output transient is the same as the output voltage produced by a unit-step in the equivalent low-pass filter. In this note it will be shown that this statement is generally incorrect and that the assumption of a narrow band is essential to validate the transient band-pass/low-pass analogy.

First, the simplest possible case will be discussed. The most rudimentary low-pass filter is a single coil L working between two resistances R. The transients are found by solving the symbolic equation $Lp + 2R = 0$ which gives the single root $p_1 = -\frac{2R}{L}$ corresponding to a transient $\exp(p_1 t)$. The low-pass is transformed into a band-pass by adding a series capacitor $C = \frac{1}{\omega_0^2 L}$ and the new new symbolic equation has two roots $\frac{1}{2}(p_1 \pm \sqrt{p_1^2 - 4/\omega_0^2})$. If p_1 can be neglected with respect to $2\omega_0$, the roots are approximately $\frac{1}{2}p_1 \pm j\omega_0$ and the corresponding two terms are combined into $\exp(\frac{1}{2}p_1 t)\cos\omega_0 t$, thus having an envelope identical to the low-pass transient. On the contrary, for small values of ω_0, both roots are real and the result bears no resemblance to the first case; no envelope can even be defined since the transient has no oscillatory character. Obviously the condition $2\omega_0 \gg |p_1|$ corresponds to a high Q for the series resonant circuit $L,C,2R$; i.e., to a narrow band.

Similar considerations hold for the general case. Let us suppose that the steady-state characteristic of a low-pass is defined by giving the output voltage $\frac{1}{S(p)}\exp(pt)$ corresponding to a generator $\exp(pt)$ at the input. The response to a unit-step will be given by Heaviside's Expansion Theorem

$$V(t) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{e^{pt}\,dp}{p\,S(p)} = \frac{1}{S(0)} + \sum_n \frac{e^{p_n t}}{p_n S'(p_n)} \quad (1)$$

where the sum extends to all the roots of $S(p)$. Since a low-pass has usually zero-loss and phase at zero-frequency, the result will take the form

$$V(t) = \tfrac{1}{2} + \sum_n A_n e^{p_n t} \quad .. \quad .. \quad .. \quad (2)$$

the p_n and the A_n being real or occurring in complex conjugate pairs.

Suppose now that a band-pass is obtained from the low-pass by the classical frequency transformation as described by Emms. The steady-state characteristic of the band-pass will be

$$T(p) = S\left(\frac{p^2 + \omega_0^2}{p}\right)$$

and the response to the voltage $\cos\omega_0 t \,.\, \mathbf{1}$ is calculated by using its Laplace transform as correctly given by Emms. The response is

$$V(t) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{e^{pt}\,p\,dp}{(p^2 + \omega_0^2)\,T(p)} \quad .. \quad .. \quad (4)$$

Thus (4) is deduced from (1) by applying the same change of variable to the factor p and in the function $S(p)$ [see (3)], but this does not correspond to any simple transformation from U to V, because the factor $e^{pt}dp$ and the path of integration are preserved.

The expansion formula will now be applied to (4). First the poles $\pm|j\omega_0|$ of the integrand give the steady-state term. The remaining poles are zeros $\hat{p}_n$ of $T(p)$ and are deduced from the zeros p_n of $S(p)$ by solving the transformation equation

$$\frac{\hat{p}_n^2 + \omega_0^2}{\hat{p}_n} = p_n$$

This gives two roots $\hat{p}_n = \frac{1}{2}\left[p_n \pm \sqrt{p_n^2 - 4\omega_0^2}\right]$ of T corresponding to each root p_n of S. The residues of T are

$$T'(\hat{p}_n) = S'(p_n)\frac{\hat{p}_n^2 - \omega_0^2}{\hat{p}_n^2}$$

the last factor being the derivative $d\hat{p}_n/dp_n$. Combining the two terms corresponding to each term of the low-pass, one obtains the result

* This article is itself an abstract of a report issued by Combined Intelligence Objectives Sub-Committee.—ED.

$$V(t) = \tfrac{1}{2}\cos\omega_0 t + \sum_n A_n \exp(\tfrac{1}{2}p_n t)$$
$$\left[\tfrac{1}{2}\left(1 + \frac{p_n}{\sqrt{p_n^2 - 4\omega_0^2}}\right)\exp\left[\tfrac{1}{2}t\sqrt{p_n^2 - 4\omega_0^2}\right]\right.$$
$$\left. + \tfrac{1}{2}\left(1 - \frac{p_n}{\sqrt{p_n^2 - 4\omega_0^2}}\right)\right.$$
$$\left.\exp\left[-\tfrac{1}{2}t\sqrt{p_n^2 - 4\omega_0^2}\right]\right] \quad .. \quad .. \quad (5)$$

Thus, if the low-pass transient is given by (2) the band-pass transient can immediately be calculated by (5). But, since in the general case the p_n's are complex, the separation of the real and the imaginary parts of the square roots, necessary to write (5) in a manageable form, will involve double radicals. Even for the single-section filter considered by Tucker, the final formulae are prohibitively cumbersome.

If all the zeros p_n of the low-pass function can be neglected as compared with $2\omega_0$, the expression between brackets is reduced to $\frac{1}{2}(e^{j\omega_0 t} + e^{-j\omega_0 t}) = \cos\omega_0 t$ and (5) becomes

$$V(t) = \cos\omega_0 t\left(\tfrac{1}{2} + \sum_n A_n e^{\frac{1}{2}p_n t}\right) = \cos\omega_0 t\, U\left(\frac{t}{2}\right)$$
$$.. \quad .. \quad .. \quad .. \quad .. \quad (6)$$

In this case the envelope of $V(t)$ is actually the low-pass transient as given by (2). Since the roots p_n of the low-pass are proportional to its cut-off frequency ω_1 and usually such that the values of p_n are not very different from ω_1 [for instance in the case of a single section filter $p_1 = -\omega_1/m$; $p_{2,3} = \frac{1}{2}\omega_1(-m \pm j\sqrt{4 - m^2})$]; the validity of (6) requires $\omega_0 \gg \omega_1$. Since the transformation (3) preserves the absolute bandwidth, this is equivalent to the condition of a narrow band.

Attention should be paid to the replacement of t by $\frac{1}{2}t$ in the envelope. This shows that the time of subsidence of the transients is twice as large in a narrow band-pass as in a low-pass having the same absolute bandwidth.

Referring to C. C. Eaglesfield's remark[1] on the discrepancy between his correct theoretical result and the experiments of D. G. Tucker[2], it should be pointed out that "physical instinct" can be deceptive for narrow band-pass filters. A band-filter with $n = 0.037$ and $Q = 100$ is the analogue of a low-pass having a Q of $\frac{1}{2}\,100 \times 0.037 = 1.85$.

V. BELEVITCH, Dr.-Ing.

Brussels.

[1] *Wireless Engineer*, November, 1946, p. 306.
[2] *Wireless Engineer*, March, 1946, p. 84.

To the Editor, "Wireless Engineer"

SIR,—The letter of E. T. Emms in the January issue seems to clear up an interesting problem. It only needs the working out from first principles of a simple example, however, to show that the statement "if the wave $\cos\omega_0 t$. **1** is put into a band-pass network then the envelope of the output wave is exactly the same as the output wave obtained when unit step is placed into the low-pass analogue" is not true.

Take for example a low-pass system of bandwidth ω_1, whose unit-step response is $1 - e^{-\omega_1 t}$. The corresponding Laplace transform is $\omega_1/(p + \omega_1)$. Hence the Laplace transform of the band-pass analogue, with applied function $\cos\omega_0 t$. **1**, is:

$$\frac{\omega_1 p}{p^2 + \omega_1 p + \omega_0^2} \cdot \frac{p^2}{p^2 + \omega_0^2}.$$

This may be interpreted by routine methods to give the corresponding unit-step response:

$$\cos\omega_0 t - \frac{e^{-\frac{1}{2}\omega_1 t}}{\sqrt{(1 - \omega_1^2/4\omega_0^2)}} \times$$
$$\cos\left[\omega_0 t\sqrt{(1 - \omega_1^2/4\omega_0^2)} + \tan^{-1}[\omega/\{2\omega_0\sqrt{(1 - \omega_1^2/4\omega_0^2)}\}]\right].$$

There are two remarks to be made about this expression: first, and mainly, it does not represent a wave whose envelope is $1 - e^{-\omega_1 t}$; if $\omega_1^2/4\omega_0^2$ can be neglected, then we do get a simple envelope, namely $1 - e^{-\frac{1}{2}\omega_1 t}$. This brings us to the second point; in order to get the band-pass analogue, from the transient point of view, of a low-pass system, we must not only replace ω by $\omega(1 - \omega_0^2/\omega^2)$ but we must also double the bandwidth. There is of course nothing new in this; an example will be found in "Communication Networks, Vol. II Chap. XI, by E. A. Guillemin. In this connection, I disagree with E. T. Emms' result for the unit-step response of the low-pass filter. In his expression, $\frac{1}{2}\omega_1$ should be replaced by ω_1.

Now look at the matter from the Fourier Integral point of view. If $g(\omega)$ is the amplitude spectrum of a time-function $f(t)$, then the amplitude spectrum of $f(t)\cos\omega_0 t$ is, as is well known,

$$\tfrac{1}{2}\{g(\omega - \omega_0) + g(\omega + \omega_0)\}.$$

Emms' statement is thus equivalent to the statement that

$$g(\omega - \omega_0^2/\omega) = \tfrac{1}{2}\{g(\omega - \omega_0) + g(\omega + \omega_2)\},$$

which, if true, is certainly not obvious. It is not, in fact, true in general; it is not true, in particular, for the function $\omega_1/2\pi j\omega(j\omega + \omega_1)$ which is the appropriate function in the example above. E. T. Emms has in effect shown that it is true for the function $1/2\pi j\omega$. But to draw from this fact the conclusion that it is true for all functions is not justified.

W. E. THOMSON.

Wembley, Middx.

REFERENCES

D. G. Tucker: "Transient Response of Filters," *Wireless Engineer*, March, 1946.
C. C. Eaglesfield: "Transient Response of Filters," *Wireless Engineer*, November, 1946.
E. T. Emms: "Transient Response of Filters" (letter), *Wireless Engineer*, January, 1947.

Use of Analogies

To the Editor, "Wireless Engineer"

SIR,—With reference to Dr. Howe's Editorial on the use of analogies (Jan., 1947), it is evident that this practice has previously come under suspicion.

Over a hundred years ago, Gay-Lussac was prompted to make use of strained analogies between hydrogen compounds and alkalis.

Criticising these, Humphry Davy remarked:—

"The substitution of analogy for fact is the bane of chemical philosophy; the legitimate use of analogy is to connect facts together, and to guide to new experiments."

These words are as true to-day as they were a century ago.

F. BUTLER.

Putney, S.W.15.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

DIRECTIONAL AND NAVIGATIONAL SYSTEMS

578 094.—Blind-landing system in which spaced receivers on the ground analyse signals sent from the aircraft; or in which a receiver on the craft indicates the phase-difference between two ground transmitters.

The General Electric Co. Ltd. and D. C. Espley. Application date 26th May, 1943.

578 275.—Radiolocation equipment, carried by an aircraft, and utilized either to locate and pursue an enemy craft, or to home on to a landing ground.

Standard Telephones and Cables, Ltd. (communicated by International Standard Electric Corporation). Application date 4th June, 1943.

578 301.—D.F. installation for indicating simultaneously the location of all transmitters operating within a wide band of frequencies, and for selecting a particular station on which to take bearings.

Standard Telephones and Cables, Ltd. (communicated by International Standard Electric Corporation). Application date 31st July, 1942.

578 302.—The use of a superposed frequency-modulation to prevent distortion, due to undesired reflection, in a course-indicator of the overlapping-beam type.

Standard Telephones and Cables, Ltd., and H. P. Williams. Application date 31st July, 1942.

578 406.—Method of modulation-control, particularly applicable to altimeters and radiolocation equipment using a frequency-modulated carrier.

H. F. Rost, K. H. Thunell, S. D. Vigren and P. H. E. Claesson. Application date 24th November, 1942.

RECEIVING CIRCUITS AND APPARATUS

577 817.—D.C. generator, fitted with an auxiliary a.c. slip-ring and rectifier circuit, for supplying all the required operating-voltages to a cathode-ray tube.

A. D. Blumlein and E. A. Nind. Application date 13th October, 1939.

578 013.—Preparation of the silicon element in a low-capacitance catswhisker combination for rectifying very high frequencies.

The General Electric Co. Ltd. and C. E. Ransley. Application date 22nd March, 1943.

578 114.—Application of a phase-shifting or time-delay device to eliminate residual noise in a receiver fitted with certain known types of a.v.c. or a.t.c.

Standard Telephones and Cables, Ltd. (assignees of C. B. H. Feldman). Convention date (U.S.A.) 28th December, 1939.

578 116.—Composition and processing of the silicon element in a catswhisker combination for rectifying very short waves.

The General Electric Co. Ltd., D. E. Jones, C. E. Ransley, J. W. Ryde and S. V. Williams. Application date 18th July, 1941.

578 201.—Device for limiting the power consumed by a police-car or like receiver which is normally kept for long periods under "stand-by" conditions.

The British Thomson-Houston Co. Ltd. Convention date (U.S.A.) 22nd January, 1943.

TELEVISION CIRCUITS AND APPARATUS
FOR TRANSMISSION AND RECEPTION

578 092.—Coloured-television system in which the colour filter for the photo-sensitive screen is vibrated in synchronism with the frame-scanning voltage.

Farnsworth Television and Radio Corporation. Convention date (U.S.A.) 14th April, 1942.

578 108.—Colour-filter designed to facilitate the viewing of monochromatic television pictures in daylight, or under bright artificial lighting.

J. L. Baird. Application date 25th April, 1944.

578 423.—Colour-television system of the kind in which the transparency of a sensitive surface is controlled by a modulated beam of electrons, and in which the picture is projected on to an external viewing-screen.

Scophony Ltd. and G. Wikkenhauser. Application date 1st May, 1944.

TRANSMITTING CIRCUITS AND APPARATUS
See also under Television

577 842.—Wave-guide comprising a circular section, with rectangular input and output sections, for securing polarization effects.

Western Electric Co. Inc. Convention date (U.S.A.) 23rd December, 1942.

577 942.—Delay-network for generating substantially flat-topped pulses, which do not cause modulation-drift, when fed to a magnetron oscillator.

D. Blumlein (legal representative of A. D. Blumlein) Application date 28th October, 1941.

578 088.—Push-pull short-wave oscillator coupled to a hollow resonator or tank circuit through a link which automatically inhibits undesired or parasitic frequencies.

The General Electric Co. Ltd. and D. C. Espley. Application date 20th May, 1943.

578 151.—Oscillation-generator, with a two-stage crystal-control and heterodyne circuits, for transmitting signals on any selected one of a number of carrier-frequencies.

Haveltine Corporation (assignees of D. E. Harnett). Convention date (U.S.A.) 30th September, 1940.

578 416.—Scanning and synchronizing system for transmitting and receiving pictures and messages in facsimile.

Creed & Co. Ltd. (assignees of S. Khalil). Convention date (U.S.A.) 29th January, 1943.

578 419.—Frequency-stabilizing device, particularly for a short-wave transmitter, in which the control voltage is developed by two velocity-modulating tubes arranged in opposition (divided from 578 406).

H. F. Rost, K. H. Thunell, S. D. Vigren and P. H. E. Claesson. Application date 24th November, 1942.

578 432.—Metering device for monitoring the performance of a valve-oscillator for generating frequency-modulated signals.

Marconi Instruments Ltd. and C. F. Brocklesby. Application date 27th July, 1944.

CONSTRUCTION OF ELECTRONIC-DISCHARGE DEVICES

578 117.—Arrangement and mounting of the " gun " structure of a short-wave oscillator wherein the electron stream is projected through a resonator.

E. K. Cole Ltd. and F. W. O. Kennedy. Application date 5th November, 1941.

578 270.—Velocity-modulating tube, comprising two or more resonators, arranged in series and tuned to harmonic frequencies, for use as a frequency-doubler.

Standard Telephones and Cables, Ltd. (communicated by International Telephone Development Co. Inc.). Application date 10th April, 1942.

578 271.—Oscillation-generator in which an electron beam is first " bunched " and is then " multiplied " by passing through a series of secondary-emission electrodes.

Standard Telephones and Cables, Ltd. (assignees of R. V. L. Hartley and C. V. Parker). Application date 30th April, 1942.

578 582.—C. R. tube fitted with a fluorescent screen having two superposed coatings, one giving a quick response, the other a more lasting after-glow.

A. C. Cossor Ltd., F. M. Walker and E. E. Shelton. Application date 13th July, 1939.

578 586.—Velocity-modulating tube in which the electron-stream is fed to the resonator through an open-ended length of a waveguide of restricted cross-section.

Standard Telephones and Cables Ltd. (Assignees of W. Shockley). Convention date (U.S.A.) 11th July, 1941.

578 587.—Velocity-modulating tube in which the final collector or anode is coated and designed to minimize undesirable secondary-emission.

Westinghouse Electrical International Co. Convention date (U.S.A.) 13th August, 1941.

578 588.—Velocity-modulating tube wherein the electron-stream is forced to pass from the " buncher " to the " catcher " resonator in a curved path so as to minimize space-charge effects.

Westinghouse Electric International Co. Convention date (U.S.A.) 20th August, 1941.

578 618.—Method of welding the resonator electrodes to the main walls of an electron-discharge tube of the velocity-modulating type.

The M-O Valve Co. Ltd., N. L. Harris and J. W. Ryde. Application dates 18th April and 31st May, 1940.

578 619.—Welding the resonators to the walls of a velocity-modulating tube in which the spacing or tuning of the elements can be adjusted (divided out of 578 618).

The M-O Valve Co. Ltd., N. L. Harris and J. W. Ryde. Application date 18th April, 1940.

578 620.—Velocity-modulating tube which is designed (a) to be free from critical spacings affecting the frequency, and (b) to employ secondary emission to increase the gain.

Standard Telephones and Cables Ltd., (assignees of W. McH. Goodall) Convention date (U.S.A.) 7th January, 1941.

SUBSIDIARY APPARATUS AND MATERIALS

577 953.—Variable control-network for stabilizing the operation of a high-frequency oscillator when used, say, for diathermy, or for spot-welding plywood.

The General Electric Co. Ltd. and L. C. Stenning. Application date 23rd October, 1942.

578 113.—Photo-electric cell in which secondary emission from the anode is utilized to ensure a predetermined voltage-response to a sudden small change of illumination.

J. D. McGee, L. Klatzow and R. E. Spencer. Application date 8th August, 1940.

578 129.—Multivibrator circuit arranged to generate short pulses (a) without the use of a delay circuit, or (b) independently of the duration of a saw-toothed triggering-voltage.

F. W. Cutts. Application date 31st January, 1944.

578 135.—Saw-tooth oscillation-generator wherein a point in the anode circuit, say of a pentode, is connected through a differentiating network to a point of fixed potential, the input voltage being derived from the said network.

A. C. Cossor Ltd. and B. C. Fleming-Williams. Application date 19th April, 1944.

578 290.—Stabilizing the operation of a crystal-oscillator when carried by V-shaped wire mountings.

The General Electric Co. Ltd., L. A. Thomas and A. H. Morser. Application date 20th October, 1944.

578 407.—Inverter circuit, comprising a saturated inductance for generating a.c. of constant magnitude from a variable source of d.c.

Electronic Laboratories Inc. Convention date (U.S.A.) 8th May, 1942.

578 456.—Valveholder comprising means for supporting a screening-can.

Carr Fastener Co. Ltd. and G. Wagstaff. Application date 27th April, 1944.

578 461.—Valve installation for generating a readily-adjustable polyphase supply, suitable for testing and research purposes, each oscillator-stage being coupled to a power-stage of the cathode-follower type.

Westinghouse Brake and Signal Co. Ltd. and A. H. B. Walker. Application date 23rd May, 1944.

578 487.—Construction and terminal connections for an electrolytic capacitor comprising interleaved sheets of metal-foil.

P. A. Sporing and The Telegraph Condenser Co. Ltd. Application date 27th March, 1944.

578 729.—Sound-ranging equipment comprising phase and amplitude control networks feeding a cathode-ray tube which gives a direct indication of the direction of the source of sound.

G. E. Condliffe and H. A. M. Clark. Application dates 3rd and 15th June, 19th July, and 10th September, 1938.